sTeX Language and IDE Tutorial

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If you have questions or problems with sTEX, you can talk to us directly at *https://matrix.to/#/#stex:fau.de*.

The dynamic HTML version of this document can be found at *https://stexmmt.mathhub.info/:sTeX/fullhtml?archive=sTeX/Documentation&filepath=tutorial.en.xhtml*

sTEX is a system for generating human-oriented documents in either PDF or HTML format, augmented with computer-actionable semantic information (conceptually) based on the OMDoc format and ontology.

In this document, we will give a broad but shallow introduction to sTEX, and what you can get out of it. Additionally, this serves as an introduction to the sTEX IDE.

Note that in PDFs, the specific highlighting of semantically annotated text is fully customizable (see chapter 9 (User Manual) in the sTEX Documentation). In this document, we use this highlighting for notation components, this highlighting for symbol references, this highlighting for (local) variables and **this highlighting** for definienda; i.e. new concepts being introduced.

Contents

0.1 This Tutorial: Overview

This tutorial has three parts: The first (**??**) introduces the foundations of semantic markup in sTeX, the second (**??**) adds functionalities that are specific to highly mathematical subjects, and the third (**??**) introduces facilities for using sTeX-based markup in educational settings.

Chapter 1

Setting Up the sTeX IDE

sTeX is based on LaTeX, and adds additional layers of presentational and functional markup to it. As a consequence the source files of sTeX documents look quite different from the resulting XHTML and PDF documents. Thus the best way of interacting the sTeX document collections is via an integrated development environment (IDE). In this tutorial we will use the sTeX plugin for the VS Code, which you should set up as a first step (this also sets up the necessary auxiliary software).

Setting up sTeX with the dedicated IDE is easy:

1. Download and install VS Code here: `https://code.visualstudio.com/download`
2. Start VS Code and navigate to the *Extensions*-tab on the left. Here you can search for Extensions in the VS Code marketplace. Look for the sTeX extension by *KWARC*, as in Figure 1.1 on the left.
3. Having done so, upon opening any folder in VS Code containing a `.tex`-file the setup window will pop up, as in Figure 1.1 on the right.

   The IDE will attempt to determine your Java installation and your `MathHub` directory (if set via an environment variable). Alternatively, you can set the latter now.
4. Download the MMT `.jar`-file at the link provided in the setup and select it. The IDE should then be able to determine your MMT version.

And that's it. Click on *Finish* and your setup is finished. The extension will start and download RusTeX and some fundamental math archives for you automatically (an internet connection is required when finishing the setup).

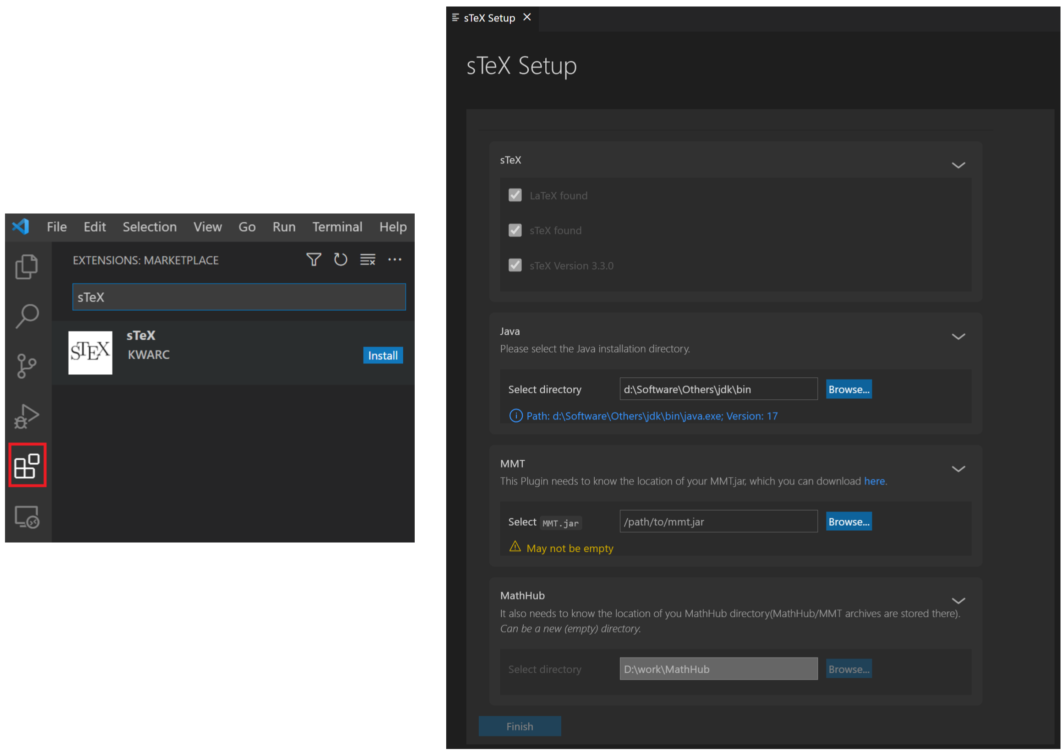

Figure 1.1: Installing the sTEX IDE

Part I

The Basics

This document itself uses sTeX and serves as a direct example for the following. You can download its source files, the generated PDF files, and the generated HTML documents directly from within the IDE, by navigating to the sTeX tab in the menu on the left and finding `sTeX/Documentation` in the list of math archives and clicking the small "Install"-button next to it, see the screenshot on the left of Figure 1.2.

Once downloading is finished (this may take a while since dependencies are also downloaded), you can then browse the `.tex`-files in `sTeX/Documentation` directly from the math archives panel in the sTeX tab, as you can see in the right screenshot in Figure 1.2.

For example, you can now navigate to the file `tutorial/intro.en` to see the sources of this very part.

As a first example, consider the following document fragment from section 1.1 (What is sTeX?) in the sTeX Documentation:

> sTeX is a system for generating human-oriented documents in either PDF or HTML format, augmented with computer-actionable semantic information (conceptually) based on the OMDoc format and ontology.

If you were to look at the generated HTML from this fragment, you could hover over the highlighted words (sTeX, PDF, HTML, OMDoc) and get a little popup with their definitions (Figure 1.3). Neat, huh?

Here, in the PDF, hovering will only show you a unique identifier (MMT-URI) for the word, and link to a definition on the web. Still useful, but not quite as neat, of course.

A plain LaTeX-version of the above document fragment, without any sTeX markup, could look like this:

Example 1

Input:

File [sTeX/Documentation]tutorial/intro/intro1plain.en.tex

```
\documentclass{article}
\usepackage{stex-logo}
\begin{document}

  \sTeX{} is a system for generating human-oriented documents
  in either \textsf{PDF} or \textsf{HTML} format, augmented
  with computer-actionable semantic information (conceptually)
  based on the \textsc{OMDoc} format and ontology.

\end{document}
```

Output:

> sTeX is a system for generating human-oriented documents in either PDF or HTML format, augmented with computer-actionable semantic information (conceptually) based on the OMDoc format and ontology.

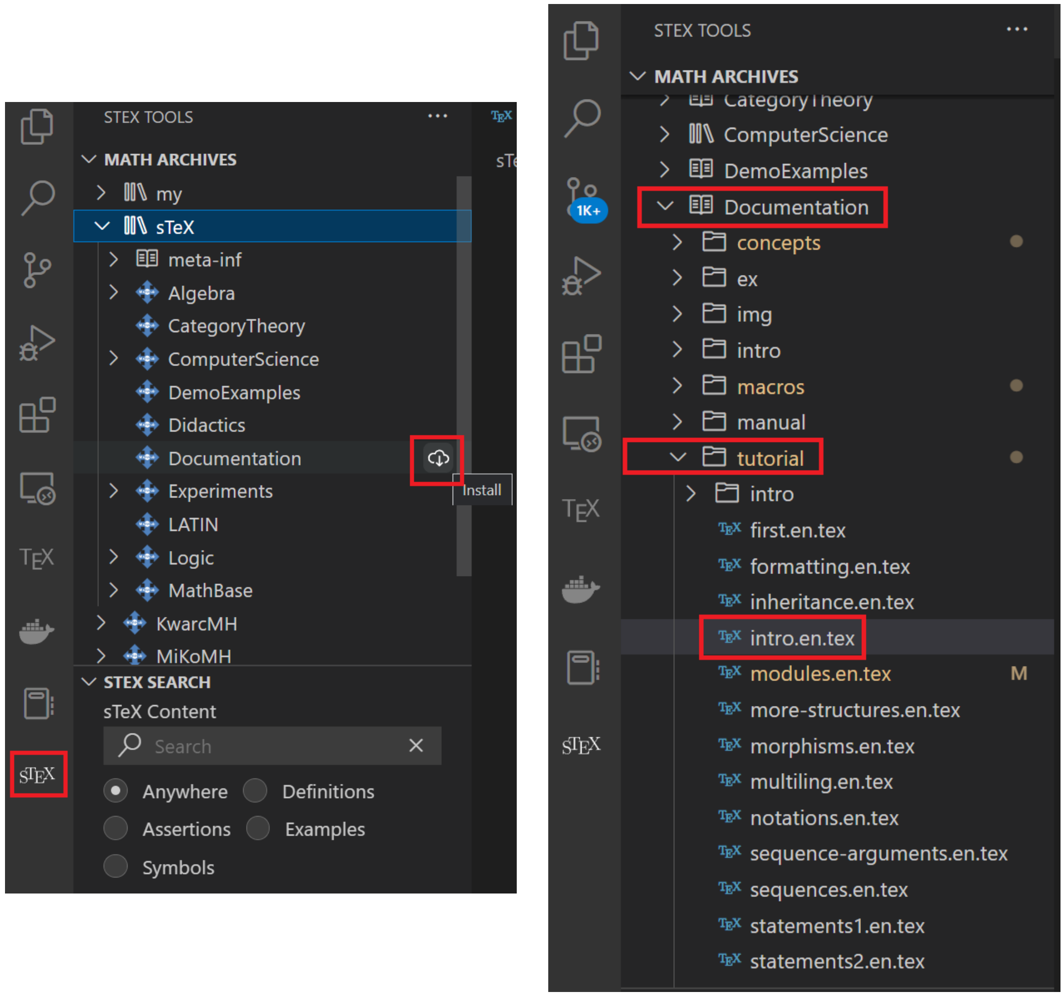

Figure 1.2: Installing Math Archives

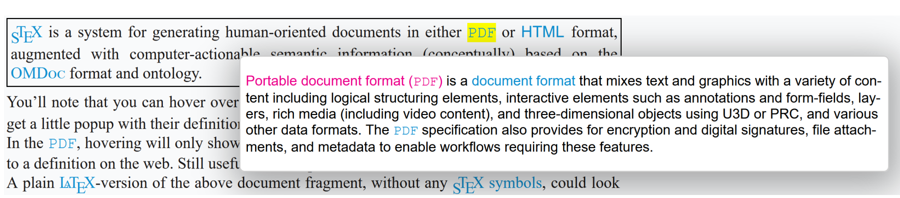

Figure 1.3: Definition on Hover

(Examples like the one above always show the file the source code is in, so if you have downloaded the `sTeX/Documentation` math archive you can toy around with it yourself)

If you save a file in the IDE (regardless of whether it has unsaved changes), a preview window will pop up, showing you the HTML generated from the `.tex`-file; see (Figure 1.4).

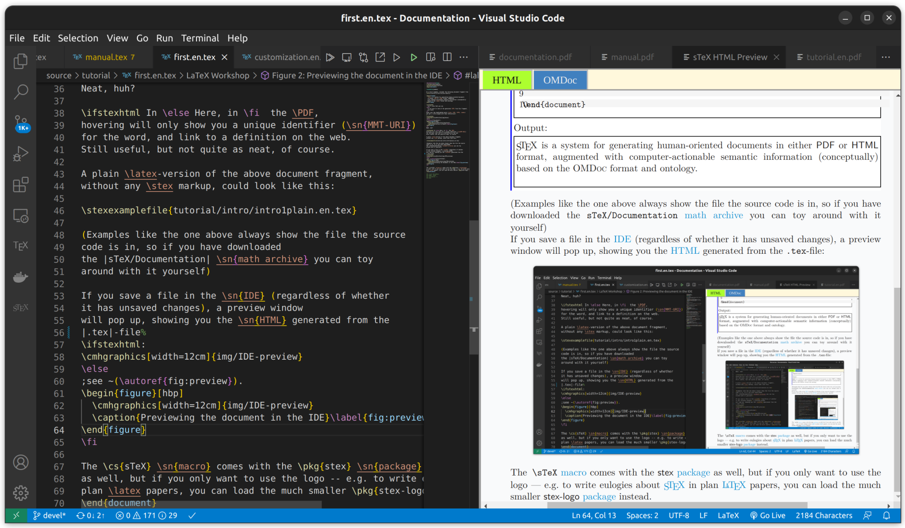

Figure 1.4: Previewing the document in the IDE

The `\sTeX` macro comes with the stex package as well, but if you only want to use the logo – e.g. to write eulogies about sTeX in plan LaTeX papers, you can load the much smaller stex-logo package instead.

Chapter 2

Text symbols

The most central concept behind sTeX is that of a *symbol*:

> sTeX
>
> A **symbol** is a *named* concept that can be defined, documented and referenced. Examples for symbols are mathematical constants, functions, theorems, statements, principles – anything that has a (somewhat) precise meaning and can be referenced by name can be a symbol.

Before we explain how we can declare new symbols and associate them with definitions, notations and all that, let's assume an ideal world in which others have done that job already for us – after all, sTeX is all about *reuse*, and naturally, there are sTeX symbols for all of the above already. Let's start with the one for sTeX itself:

2.1 Using Modules & Search in the IDE

In the VS Code IDE, navigate to the sTeX-tab on the left. In the search panel, select the "`Symbols`" radio button and search for "`sTeX`". The second search result should be what we're looking for (Figure 2.1).

Search results are grouped into *local* and *remote* results. Local ones are the ones you already have in your local `MathHub` directory; remote ones you can download directly from within the IDE.

You can click the preview button to see the generated HTML for the document – the resulting window that pops up also has an OMDoc tab you can select, which (among other things) shows you the semantic macros provided by the respective module: In this case, it tells us that there is a *text symbol* named "`sTeX`" with semantic macro `\stex` in the module `mod/systems/tex?sTeX` that is in the `\sTeX/ComputerScience/Software` archive. It produces the presentation "sTeX" as we want (Figure 2.2).

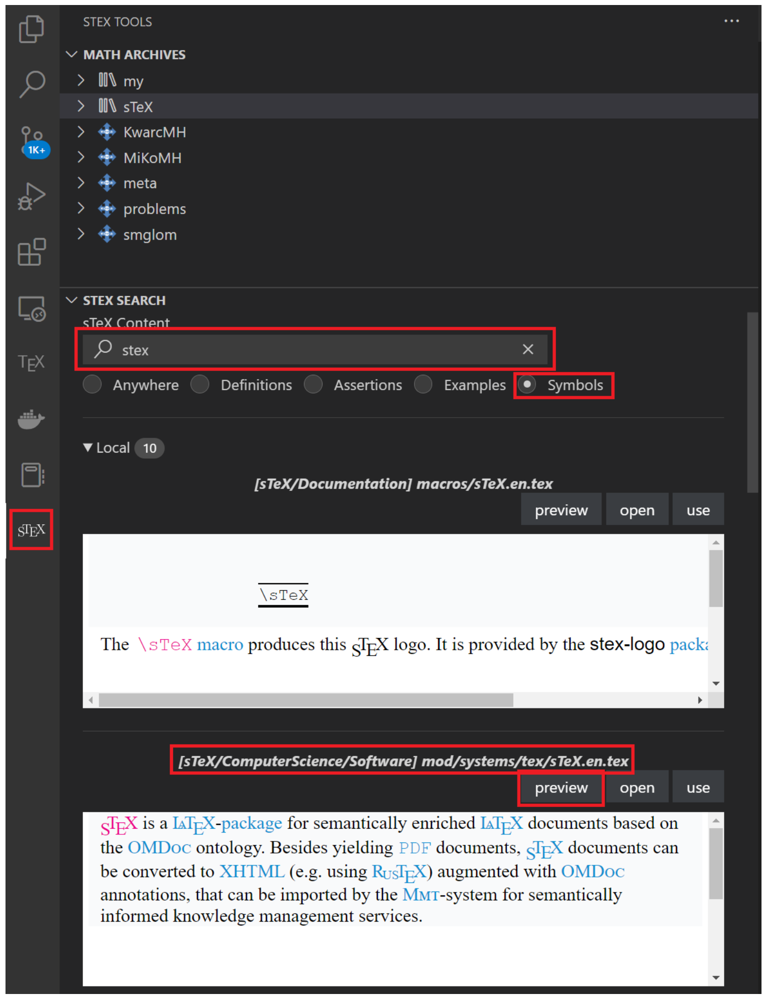

Figure 2.1: Search in the sTeX IDE

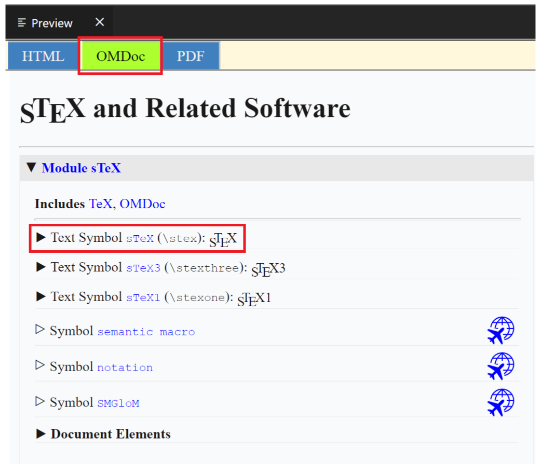

Figure 2.2: OMDoc Preview

> sTeX A **text symbol** is a symbol foo with an associated semantic macro \foo. The macro \foo is allowed in text or math mode and produces a predefined piece of text output annotated with foo.
> The variant \fooname produces the same output without annotation.

If we want to use the sTeX symbol in a document – which we have open in the IDE – we simply click on the `use` button, and the IDE will automatically insert the line `\usemodule[sTeX/ComputerScience/Software]{mod/systems/tex?sTeX}`, making all symbols in that module available to use – in particular, we can now use the `\stex` semantic macro instead of the plain, non-semantic `\sTeX` macro – that is, of course, after we include the `stex` package first.

> sTeX The `\usemodule` macro takes as *optional* argument the name of a math archive, and as a regular argument the path to an sTeX module (see section 7.5 (Simple Inheritance) in the sTeX Documentation).

Analogously, we can also search for the PDF, HTML and OMDoc symbols, all of which are also text symbols and have the associated semantic macros `\PDF`, `\HTML` and `\omdoc`; the document should thus look like this:

Example 2
Input:

File [sTeX/Documentation]tutorial/intro/intro1stex.en.tex

```
\documentclass{article}
\usepackage{stex}
\begin{document}
  \usemodule[sTeX/ComputerScience/Software]{mod/systems/tex?sTeX}
  \usemodule[sTeX/ComputerScience/Software]{mod/formats?PDF}
  \usemodule[sTeX/ComputerScience/Software]{mod/formats?HTML}
  \usemodule[sTeX/ComputerScience/Software]{mod/formats?OMDoc}

  \stex is a system for generating human-oriented documents
  in either \PDF or \HTML format, augmented
  with computer-actionable semantic information (conceptually)
  based on the \omdoc format and ontology.
\end{document}
```

Output:

> sTeX is a system for generating human-oriented documents in either PDF or HTML format, augmented with computer-actionable semantic information (conceptually) based on the OMDoc format and ontology.

Now, our generated HTML looks a lot more interesting, with highlighting, pop-ups on hover and all that. Notably however, if we compile the file with `pdflatex`, it looks pretty much exactly as before – except for (optional/configurable) colors.

That's because we haven't told sTeX what to do with semantic annotations yet – and by default, it does not do anything fancy, except for wrapping them in an `\emph`. We can customize how we want sTeX to highlight various semantic text fragments (see chapter 9 (User Manual) in the sTeX Documentation). A default highlighting schema is provided in the stex-highlighting package – including that will

- highlight semantically annotated text in this color,
- show the MMT-URI of the corresponding symbol in a tooltip on hovering over the text,
- make the text link to the place the symbol is being defined in the current document (if it is), or, alternatively,
- make it link to an external resource, if one is known. In our case, they link to `stexmmt.mathhub.info/:sTeX`, where the HTML for all the symbols we use in this document are hosted.

Note that in the IDE, the `\usemodule`-statement for OMDoc is underlined in blue (Figure 2.3) – VS Code is letting us know, that this `\usemodule` statement is *redundant*. That is because the sTeX module we imported earlier already imports the OMDoc module; as such we have all macros therein available already. If we look at the sTeX module in the VS Code preview window again, we can see that (Figure 2.4).

We can consequently safely delete the `\usemodule` again.

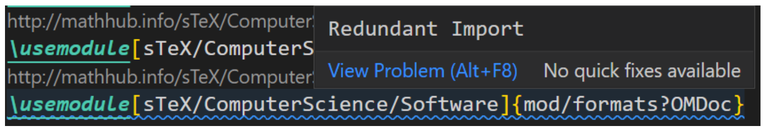

Figure 2.3: Redundant Imports

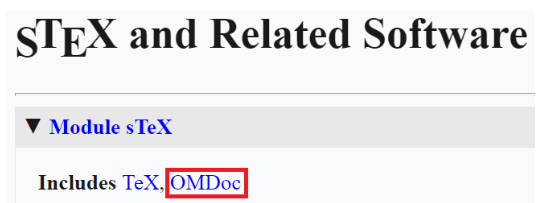

Figure 2.4: Includes in the OMDoc Preview

Chapter 3

Symbol References

Let's continue with the next paragraph of section 1.1 (What is sTeX?) in the sTeX Documentation; for now unannotated:

Example 3

Input:

File [sTeX/Documentation]tutorial/intro/intro2plain.en.tex

```
\documentclass{article}
\usepackage{stex}
\begin{document}

  At its core is the \sTeX{} package for \LaTeX, that allows for
  semantically marking up document fragments; in particular
  concepts, formulae and mathematical statements (such as
  definitions, theorems and proofs). Running \texttt{pdflatex}
  over \sTeX-annotated documents formats them into normal-looking
  \textsf{PDF}.

\end{document}
```

Output:

At its core is the sTeX package for LaTeX, that allows for semantically marking up document fragments; in particular concepts, formulae and mathematical statements (such as definitions, theorems and proofs). Running `pdflatex` over sTeX-annotated documents formats them into normal-looking PDF.

We already know how to annotate "sTeX" and "PDF"; and if we use the search field in the IDE again, we can also find a text symbol for "LaTeX". But if we look at the documentation, we will note that *more* is highlighted:

At its core is the sTeX package for LaTeX, that allows for semantically marking up document fragments; in particular concepts, formulae and mathematical statements (such as definitions, theorems and proofs). Running `pdflatex` over sTeX-annotated documents formats them into normal-looking PDF.

The "`package`"-symbol can be found in the LaTeX module too, and searching for the keywords "formula" and "mathematics" will yield the symbols "`well-formed formula`" and "`mathematics`", but they are not *text symbols* and "`mathematics`" and "`package`" do not even have a semantic macro – and the one for "`well-formed formula`" would not work outside of math mode.

Text symbols are special in that way – they are intended for symbols that have a specific formatting associated (such as LaTeX, OMDoc, or HTML, which we prefer to typeset as sans serif). For those settings, it makes sense to associate that formatting with a semantic macro that does the typesetting for us.

Symbols *without* a text macro can be referenced with the `\symname` macro: `\symname{package}` prints the *name* of the "`package`"-symbol and annotates it accordingly, without any special formatting – in particular it is compatible with being in `\emph`, `\textbf` and similar macros. That takes care of *one* of the missing annotations.

More generally, the `\symref` macro can be used to annotate arbitrary text with a symbol: `\symref{mathematics}{mathematical}` associates the text `mathematical` with the symbol "`mathematics`"; thus, we get "mathematical" and similarly "formulae".

sTeX

In general, any macro that expects a symbol name can be given either

1. the *name* of the symbol,
2. the name of its semantic macro,
3. or any suffix of its MMT-URI containing at least the module name.

The second option is often short – and therefore convenient to write; for example, to achieve "formulae", we can also write `\symref{wff}{formulae}`, since `\wff` is the semantic macro for "`well-formed formula`".
The third option allows for distinguishing between multiple symbols with the same name – the IDE can help in the latter case, by underlining ambiguous symbol references in yellow, and offering the `Quick Fix` functionality to let you select and autocomplete the specific symbol you want to reference.

Since `\symname` and `\symref` are a lot to type for something that should ideally be used as often as possible, the macros `\sn` and `\sr` exist as well and behave exactly the same way. We also provide some convenience abbreviations for `\sn`; namely `\Sn` (capitalizes the first letter of the symbol name), `\sns` (adds an "`s`" at the end, for the most common pluralization of a name), and `\Sns` (both).

Using all of the above, our annotated fragment now looks like this:

Example 4
Input:

File [sTeX/Documentation]tutorial/intro/intro2stex.en.tex

```
\usemodule[sTeX/ComputerScience/Software]{mod/systems/tex?sTeX}
\usemodule[sTeX/Logic/General]{mod/syntax?Formula}
\usemodule[sTeX/MathBase/General]{mod?Mathematics}
\usemodule[sTeX/ComputerScience/Software]{mod/formats?PDF}

  At its core is the \stex \sn{package} for \latex, that allows for
  semantically marking up document fragments; in particular
  concepts, \sr{wff}{formulae} and \sr{mathematics}{mathematical}
  statements (such as definitions, theorems and proofs). Running
  \texttt{pdflatex} over \stex-annotated documents formats them
  into normal-looking \PDF.
```

Output:

At its core is the sTeX package for LaTeX, that allows for semantically marking up document fragments; in particular concepts, formulae and mathematical statements (such as definitions, theorems and proofs). Running `pdflatex` over sTeX-annotated documents formats them into normal-looking PDF.

There's only one problem: *the document does not compile*, with an error `Undefined control sequence`. The reason being that *some* macro in the module `Formula` uses the `\text` macro. We can fix that by using the amsfonts package of course, but this points to a more general problem; namely that modules can make use of various LaTeX packages for typesetting symbols.

Good practice suggests putting those packages into a *prelude* per math archive, which we can then import from anywhere, using the `\libinput` macro. For more on that, see section 5.3 (The `lib`-Directory) in the sTeX Documentation.

For now, suffice it to say that we can import all packages required for the module `Formula` from the math archive `sTeX/Logic/General` by adding the line

```
\libinput[sTeX/Logic/General]{preamble}
```

before the `\begin{document}`.

Chapter 4

Modules and Simple Symbol Declarations

Consider again the first two paragraphs of section 1.1 (What is sTeX?) in the sTeX Documentation:

> sTeX is a system for generating human-oriented documents in either PDF or HTML format, augmented with computer-actionable semantic information (conceptually) based on the OMDoc format and ontology.
>
> At its core is the sTeX package for LaTeX, that allows for semantically marking up document fragments; in particular concepts, formulae and mathematical statements (such as definitions, theorems and proofs). Running `pdflatex` over sTeX-annotated documents formats them into normal-looking PDF.

Firstly, note that the first paragraph would be perfectly suitable to serve as a pop-up definition on hover for the sTeX symbol. Secondly, what if all the symbols used in the above *didn't* already exist?

In this chapter, we will describe how to make your own symbols and collect them as reusable fragments in modules and math archives from scratch.

We start by creating a new math archive. In the IDE, switch to the sTeX-tab on the left and click the "`New sTeX Archive`" button (Figure 4.1). You will then be asked for the name of the archive, a namespace for its content, and a url-base, where the content is supposedly going to end up online. You can safely keep the defaults for the latter two. In the following, we assume that your archive is named `my/archive`.

The IDE will then create the following files and directories in your MathHub directory:

```
- my
  - archive
    - lib
      - preamble.tex
    - META-INF
      - MANIFEST.MF
```

```
  - source
    - helloworld.tex
```

…and open the file `helloworld.tex` with the content

```
\documentclass{stex}
\libinput{preamble}
\begin{document}
% A first sTeX document
\end{document}
```

You can now reference any newly created content in you new archive using for example `\usemodule[my/archive]{...}`.

Let's start with the "LaTeX" symbol. Rename the file `helloworld.tex` to something more meaningful, for example `latex.en.tex` – the `.en` will be picked up on by sTeX to signify that the fragment will be in *english* (see subsection 7.1.1 (Signature Modules, Languages, and Multilinguality) in the sTeX Documentation).

What we want to achieve in this file is the following:

> **TeX** is a document typesetting software developed by Donald Knuth, with a focus on mathematical formulae. It is based on a powerful and extensible **macro** expansion engine.
>
> **LaTeX** is a (nowadays) default collection of TeX macros developed by Leslie Lamport. Among other things, LaTeX introduces **environments**, a distinction between preamble and document content, **packages** to bundle and distribute macro definitions, and **document classes**: special packages that govern the global layout of a document.

In particular, in the HTML the two paragraphs above should be shown when hovering over the symbols they define (as indicated by the magenta definiendum highlighting). So we need symbols and semantic macros, for: TeX, macro, LaTeX, environment, package and document class.

Symbol declarations are only allowed within modules:

> sTeX A **module** is a *named* block that bundles symbol declarations for subsequent reuse. A module is introduced with the `smodule`-environment.

Let's name our module `LaTeX`. We then wrap the contents of our document in a `smodule` environment:

```
\begin{document}
  \begin{smodule}{LaTeX}
    ...
  \end{smodule}
\end{document}
```

Note, that the IDE immediately picks up on this and displays the full MMT-URI of our new module over the `\begin{smodule}{LaTeX}` (Figure 4.2) –

From this, we can glimpse that the namespace of the module is `http://mathhub.info/my/archive/latex`. This implies, that to use the module somewhere else, we will have to type `\usemodule[my/archive]{latex?LaTeX}` – the `latex`-part pointing to the *file* and `LaTeX` referring to the actual module.

If we rename the file to `LaTeX.en.tex`, we notice that the namespace changes to `http://mathhub.info/my/archive`, allowing us to now use it with `\usemodule[my/archive]{LaTeX}` directly. That's because the module name `LaTeX` and the file name `LaTeX` match now (see section 7.5 (Simple Inheritance) in the sTeX Documentation, Figure 4.3).

> Note that "`LaTeX`" and "`latex`" only differ in capitalization – if your file system is case-insensitive (as e.g. MacOS's was until quite recently), this distinction gets murky, but remains very important especially if you want to share your math archive with others!
> It is therefore *highly recommended* to treat file names as case-sensitive either way.

Within the module, we can now declare new symbols using the `\symdecl`-macro. We start with those that are not text symbols:

```
\symdecl*{macro}
\symdecl*{environment}
\symdecl*{package}
\symdecl*{document class}
```

The `*` after the `\symdecl` indicates, that we do not want a semantic macro for the symbol – otherwise, it would generate one with the same name as the symbol itself and "pollute the macro space", so to speak.

The symbols TeX and LaTeX, however, have a definite way of being typeset associated with them, which can be produced using the standard `\TeX` and `\LaTeX` macros. So let's make them text symbols, using the `\textsymdecl` macro:

```
\textsymdecl{tex}{\TeX}
\textsymdecl{latex}{\LaTeX}
```

The first argument being the name of the generated macro (i.e. `\tex` and `\latex`) and the second one specifying the output to produce.

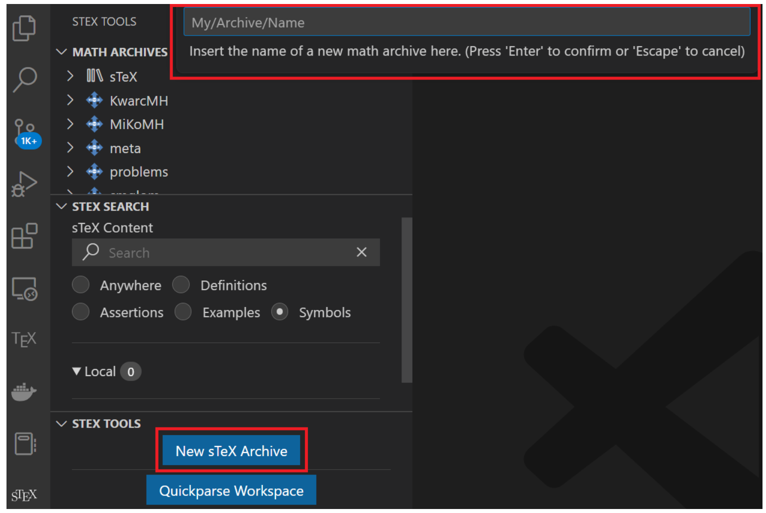

Figure 4.1: New Math Archive in the IDE

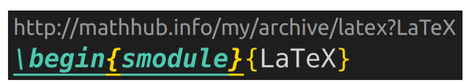

Figure 4.2: VS Code Code Lense

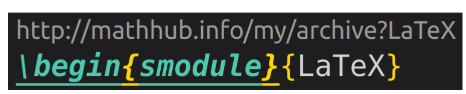

Figure 4.3: VS Code Code Lense

Chapter 5

Documenting Symbols

We can now use the two new macros, `\symname`/`\sn`, `\symref`/`\sr` etc. to mark up the above two paragraphs. But the IDE also makes us aware of the symbols not yet being documented, via squiggly blue lines(Figure 5.1).

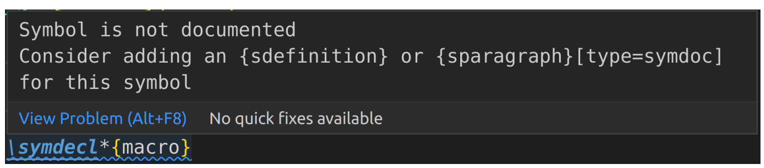

Figure 5.1: Undocumented Symbols

Among other things, this means that the system does not yet know what to show a reader when hovering over the symbol in the HTML. The IDE also recommends two ways to fix that: The `sdefinition` or `sparagraph` environments.

Ignoring the former for now, which is more useful for mathematical concepts, we can use the following to mark up the first paragraph:

```
\begin{sparagraph}[style=symdoc,for={tex,macro}]
  \tex is a document typesetting
  software developed by Donald Knuth, with a focus on
  mathematical formulae. It is based on a powerful
  and extensible \sn{macro} expansion engine.
\end{sparagraph}
```

In general, the `sparagraph` environment can be used to mark up arbitrary paragraphs semantically, but the `style=symdoc` option tells sTeX to use this paragraph as a documentation for the symbols provided in the `for=` option. And indeed, doing so makes the squiggly blue lines in the IDE under `\textsymdecl{tex}{TeX}` and `\symdecl*{macro}` disappear.

We just used the semantic macro `\stex` and the `\sn` macro to mark up the fragment – but we can do better. Both concepts are being *introduced* in the above paragraph, and we can let sTeX know that that is the case:

Within an sparagraph environment with style=symdoc (or an sdefinition environment), we can mark up *definienda*, meaning the terms *being defined*, explicitly. Analogously to \symname and \symref, we have the macros \definame and \definiendum for that purpose.

Note that the \tex macro induced by the text symbol above already marks up the "TEX" it produces, so wrapping it in another \definiendum would be redundant. However, every text symbol also generates a *second* macro with the suffix name that generates a non-marked-up version of the same presentation. In other words, we get the macro \texname for free, that produces "TEX" (of course, we could just as well use the \TeX macro, but that one you probably know already).

Furthermore, every \definiendum or \definame automatically adds the symbol being referenced to the internal for=-list of the sparagraph environment, obviating the need to list it explicitly.

As such, we can produce a better markup like this:

```
\begin{sparagraph}[style=symdoc]
  \definiendum{tex}{\texname} is a document typesetting
  software developed by Donald Knuth, with a focus on
  mathematical formulae. It is based on a powerful
  and extensible \definame{macro} expansion engine.
\end{sparagraph}
```

Exercise

In your archive my/archive, create additional files that produce the following outputs:

Mathematics.en.tex

To do **mathematics** is to be, at once, touched by fire and bound by reason. This is no contradiction. Logic forms a narrow channel through which intuition flows with vastly augmented force.

– Jordan Ellenberg

PDF.en.tex

Portable Document Format (PDF) is a document format that mixes text and graphics with a variety of content.

HTML.en.tex

The **HyperText Markup Language (HTML)** is a representation format for web-pages.

OMDoc.en.tex

OMDoc is a document format for representing mathematical documents with their flexiformal semantics.

such that the following file compiles and shows the above snippets on hover:

sTeX.en.tex

```
\documentclass{stex}
\libinput{preamble}
\begin{document}
\begin{smodule}{sTeX}
  \usemodule{OMDoc}
  \usemodule{PDF}
  \usemodule{HTML}
  \textsymdecl{stex}{\sTeX}
  \begin{sparagraph}[style=symdoc]
    \definiendum{stex}{\stexname} is a system for generating
    documents in either \PDF or \HTML format, augmented with
    computer-actionable semantic information (conceptually)
    based on the \OMDoc format and ontology.
  \end{sparagraph}
\end{smodule}
\end{document}
```

sTeX is a system for generating documents in either PDF or HTML format, augmented with computer-actionable semantic information (conceptually) based on the OMDoc format and ontology.

The preamble of every file should only be

```
\documentclass{stex}
\libinput{preamble}
```

and the macros `\OMDoc`, `\PDF`, `\HTML` should produce `\textsc{OMDoc}`, `\textsf{PDF}` and `\textsf{HTML}`, respectively (but with semantic annotations of course).

Solution: Can be found in `[sTeX/Documentation]source/tutorial/solution`

Chapter 6

Sectioning and Reusing Document Fragments

We know now how to import and reuse the symbols of some module (using `\usemodule`). What about the actual document *content*?

Assume we want to write a new article that includes all of the fragments in `my/archive` we made so far, in a file `all.en.tex` in the same math archive:

```
\documentclass{article}
\usepackage{stex}
\libinput{preamble}
\begin{document}
  \author{Me}
  \title{The \texttt{my/archive} Archive}
  \maketitle
  \tableofcontents
  ...
\end{document}
```

In there, we want sections as follows:

```
- 1 Preliminaries
  (Mathematics)
  - 1.1 Document Formats
    (PDF)
    (HTML)
    (OMDoc)
- 2 \TeX and Friends
  (LaTeX)
  (sTeX)
```

We could of course do the following:

```
\section{Preliminaries}
  \input{Mathematics.en}
  \subsection{Document Formats}
    \input{PDF.en}
```

```
    \input{HTML.en}
    \input{OMDoc.en}
  \section{\TeX and Friends}
    \input{LaTeX.en}
    \input{sTeX.en}
```

...but this approach has two drawbacks:

Firstly, we need to manually keep track of the section levels, by explicitly writing `\section`, `\subsection` etc. This is fine as long as we are just interested in this particular article. But what if we want to *reuse* the article's content in another document at some point? The section levels might be entirely different then – e.g. we might want the "Preliminaries" section to be a subsection instead.

Secondly, the `\input` macro considers the file name/path provided to be either *absolute* or relative to the *current* `tex` *file being compiled* – which means that the `\input{Mathematics.en}` only works for files in the same directory as `Mathematics.en.tex`.

In short: using `\section`, `\chapter` etc. explicitly, and `\input` to reuse fragments, breaks reusability.

Instead of using `\section` and `\subsection`, sTeX therefore provides the `sfragment` environment.

`\begin{sfragment}{Foo}...\end{sfragment}` inserts a sectioning header depending on the current section level and availability. These are: `\part`, `\chapter`, `\section`, `\subsection`, `\subsubsection`, `\paragraph` and `\subparagraph`. This allows us to do the following instead:

```
  \begin{sfragment}{Preliminaries}
    \input{Mathematics.en}
    \begin{sfragment}{Document Formats}
      \input{PDF.en}
      \input{HTML.en}
      \input{OMDoc.en}
    \end{sfragment}
  \end{sfragment}
  \begin{sfragment}{\TeX and Friends}
    \input{LaTeX.en}
    \input{sTeX.en}
  \end{sfragment}
```

The only problem remaining now is that if we do this, sTeX will insert a `\part` for the first `sfragment`. If we want the "top-level" sectioning level to be `\section` instead, we can insert a `\setsectionlevel{section}` in the preamble.

As a more reuse-friendly replacement of `\input`, sTeX provides the `\inputref` macro. Using that has two advantages: Firstly, its argument is relative to some (optionally provided, or the current) math archive and is thus independent of the specific location of the file relative to the currently being compiled `.tex`-file. Secondly, when converting to HTML, it will *not* "copy" the referenced file's content in its entirety (as `\input` would), but instead dynamically insert the already existent (if so) HTML of the referenced file, avoiding content duplication and having to process the file all over again.

In general \inputref[some/archive]{file/path} inputs the file file/path.tex in the archive some/archive. As the \input-ed files in the example above are in the same archive anyway, we can simply substitute the \inputs by \inputrefs and call it a day.

Finally, we can make two more minor changes:

1. The *title* of our document is only supposed to be there, if we compile the document directly – if we were to \inputref our file into a "driver file" all.en.tex, the title and the table of contents should be omitted.

   We can achieve this using the \ifinputref conditional: by wrapping the header in an \ifinputref **\else...\fi**, it will only be processed if the file is *not* being loaded using \inputref. \ifinputref is a "classic" TeX conditional and is treated as such in both PDF and HTML compilation. A smarter macro to use is \IfInputref, which takes two arguments for the *true* and *false* cases, respectively. Additionally, when compiling to HTML, *both* arguments to \IfInputref will be processed, and the backend will decide which of the two to present when serving a document.

2. The table of contents should also be omitted in HTML mode. To achieve that, we can use the \ifstexhtml conditional, which is *true* if the document is being compiled to HTML, and *false* if compiled to PDF.

Note, that since *both* arguments of \IfInputref are processed, they should *not* open TeX groups or environments!

In summary, we can modify our document to do the following:

```
\IfInputref{}{
  \author{Me}
  \title{The \texttt{my/archive} Archive}
  \maketitle
  \ifstexhtml \else \tableofcontents \fi
}
```

The final all.en.tex can be found in [sTeX/Documentation]tutorial/solution/all.en.tex.

Chapter 7

Building and Exporting HTML

So far we know how to write sTeX documents, (we assume) how to build PDF files from them (via `pdflatex` of course), and on saving documents the IDE will preview the generated HTML. But if we do that with our new `all.en.tex`, we get presented with Figure 7.1 Where did all of our fragments go?

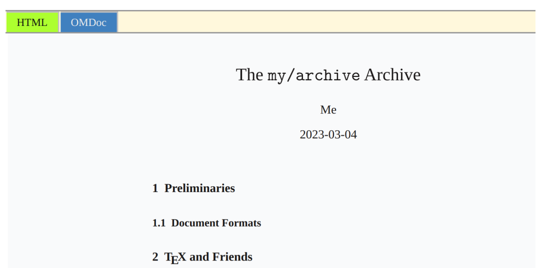

Figure 7.1: Missing Fragments in the HTML Preview

Well, they don't exist yet as HTML. The HTML Preview window in the IDE is really just that: A *preview*. But when using `\inputref`, it has to find the HTML of the `\inputref`ed fragment *somewhere*. Meaning: we have to compile all of the fragments we used to HTML first. Individually, we can compile the currently open file in VS Code using the button in Figure 7.2.

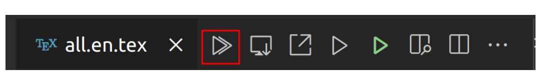

Figure 7.2: The `Build PDF/XHTML/OMDoc` Button

This will do the following:

1. Run `pdflatex` over the file three times.

2. Store the resulting .pdf in [archive]/export/pdf/<filepath>.pdf.
3. Convert the file to HTML and store it in [archive]/xhtml/<filepath>.xhtml.
4. Extract all the semantics and store them as OMDoc in [archive]/content/..., [archive]/narration/... and [archive]/relational/....
5. Construct a search index in [archive]/export/lucene/....

Doing all of this for every individual file *in hindsight* would of course be a huge hassle. We can therefore just compile the full archive, folders in an archive, or whole *groups* of archives via right-clicking an element in the Math Archives viewer in the sTEX tab (Figure 7.3).

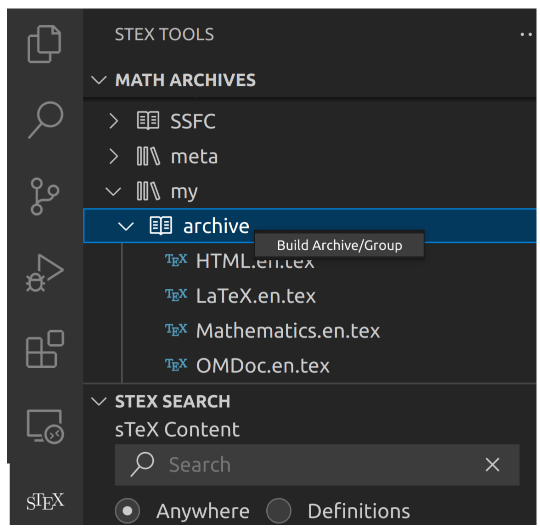

Figure 7.3: Building Archives in the IDE

Once that's done, saving all.en.tex again yields the correct HTML in the preview window.

At this point, it should be noted that you can't actually just open the HTML files exported to [archive]/xhtml in your browser and get all of the expected functionality – that shouldn't be too surprising. Features like the fancy pop-up windows require a semantically informed backend infrastructure, in the form of the Mmt system. However, Mmt *can* dump a standalone version for you. Let's do that now:

With our all.en.tex file open and everything built as above, click the Export Standalone HTM button in the IDE (see Figure 7.4).

In the dialog box that opens now, select an **empty** directory and Mmt will dump a standalone version of our all.en.tex document there. You will still not be able to open it in the browser directly, because most browser forbid javascript modules on the

Figure 7.4: Exporting HTML in the IDE

file:// protocol, but opening the file via http will yield the desired result, and you can now upload the directory's content to wherever you might want to use it.

If you want to test this, a quick and easy way to do so is to use VS Code: You can install the Live Server extension, open the directory and click the Go Live button on the lower right of the window, which will start a small web-server in the selected directory and open its index.html in the browser for you.

Part II

Mathematical Concepts

So far, we have seen how to declare and reference symbols generate semantic macros for text symbols, collect them in modules and document them properly.

But where sTEX really shines is when it comes to mathematics and related subject areas: semantic macros are significantly more useful when used for generating symbolic notations in math mode, and by associating symbols with (flexi-)formal semantics, sTEX can even *check* that your content is (to some degree) formally correct, or at least well-formed.

Alos sTEX provides specialized functionality for mathematical statements: the text fragments marked as Definition, Theorem, Proof that are iconic to mathematical documents.

The example snippets in this part can be found in the math archive `sTeX/MathTutorial`. If you downloaded the `sTeX/Documentation` archive in the sTEX IDE, you already have that archive. If not, you can download it from within the IDE, as described in Part I.

Chapter 8

Simple Symbol Declarations

We will start with symbols and semantic macros for mathematical concepts and objects and their contribution to mathematical formulae.

8.1 Semantic Macros and Notations

Let us start with a very fundamental concept; namely equality. As you should by now know, declaring a new symbol requires a module, so let's open a new one and use `\symdecl`:

```
\begin{smodule}{Equality}
  \symdecl{equal}
\end{smodule}
```

As mentioned in chapter 4, the starred variant `\symdecl*` does not create a semantic macro, so presumably, the variant without a `*` *does*. And indeed, we now have a macro `\equal`, which however will produce errors if we try to use it. That's because we haven't told sTeX what to do with it yet.

> sTeX
>
> A **semantic macro** is a LaTeX-macro that allows for referencing a symbol itself, or – in the case of e.g. a function – the *application* of a symbol to (one or multiple) *arguments*; primarily by invoking a symbol's notation in *math mode*.
> The command `\symdecl{macroname}` declares a new symbol with name `macroname` and a semantic macro `\macroname`. In the case where we want the name and the semantic macro to be distinct, the command `\symdecl{macroname}[name=some name]` declares the name of the symbol to be `some name` instead.
> The starred variant `\symdecl*{name}` declares the concept with the given name, but does not generate a semantic macro.

So let's provide equality with a notation. As a first step, we should let sTeX know that "equal" takes two arguments. We might also want to shorten the semantic macro to e.g. `\eq`, without changing the name. Hence:

```
\symdecl{eq}[name=equal,args=2]
```

Next, we add an infix notation with the `\notation` macro:

```
\notation{eq}{#1 = #2}
```

That seems like a lot to write, so for the very common case where we want to declare a symbol with a semantic macro and a notation all at once, the `\symdef` macro does all three by combining the optional and mandatory argument of `\symdecl` and `\notation`:

```
\symdef{eq}[name=equal,args=2]{#1 = #2}
```

and indeed, we can now use the `\eq` macro in math mode to invoke our new notation: `$\eq{a}{b}$` now yields $a = b$ – notably without any highlighting (and hover interaction in the HTML) though. Since our semantic macro takes *arguments*, which should be differently highlighted, we need to let our notation know which parts of the notation are highlightable components.

We can do so with the `\comp` and `\maincomp` macros:

STEX

The `\comp`-macro marks components to be highlighted in a notation for a symbol taking (one or more) arguments.
This is necessary because it is (nearly) impossible for LaTeX to figure out, which parts of a notation to highlight and which not on its own – in particular, the highlighting should stop for the *arguments* of a semantic macro.
Additionally, the `\maincomp` macro can be used to mark (at most) one notation component to represent the *primary* component of the notation.
Notations that do not take arguments, as well as operator notations, are automatically wrapped in `\maincomp`.

In our case, this applies only to the "=", symbol, so:

```
\symdef{eq}[name=equal,args=2]{#1 \mathrel{\maincomp{=}} #2}
```

You may be wondering about the role of the `\mathrel` macro in the example above: TeX determines spacing/kerning in math mode by assigning a *class* to every character. Both individual characters and whole subexpressions can be assigned one of these classes using dedicated macros. These are:

class	**TeX macro**	**examples**
ordinary (default class)	`\mathord`	$\alpha\ i\ \Diamond$
large operator	`\mathop`	$\sum \prod \int$
opening	`\mathopen`	$([\langle$
closing	`\mathclose`	$)] \rangle$
binary relation	`\mathrel`	$\leq > =$
binary operator	`\mathbin`	$+ \cdot \circ$
punctuation	`\mathpunct`	$, ;$

TeX "forgets" the class of an expression if it is wrapped in a `\comp` macro. It is therefore a good idea to wrap any occurence of a `\comp` in the corresponding TeX macro for the desired class (e.g. `\mathrel{\comp{\leq}}`).

Having done so, we can now type `$\eq{a}{b}$` to get $a = b$. Thanks to using `\maincomp`, we now also have an operator notation, which we can invoke using `$\eq!$`, yielding $=$.

What if we want to add more notations? Say we want to be able to invoke equality to get the variant notation $a \equiv b$ ()without changing the intended meaning). If we want to be able to choose one of several notations, we should give the notation an *identifier*.

Let's again modify our earlier notation by adding the identifier `eq` to the optional arguments of `\symdef`, like so:

```
\symdef{eq}[name=equal,args=2,eq]{#1 \mathrel{\maincomp{=}} #2}
```

We can now invoke the specific notation provided here by writing `$\eq[eq]{a}{b}$` to the same effect. But we can also add more notations using the `\notation` macro:

```
\notation{eq}[equiv]{#1 \mathrel{\maincomp{\equiv}} #2}
```

which we can now invoke with `$\eq[equiv]{a}{b}$`, yielding $a \equiv b$.

By default, the *first* notation provided for a given symbol is considered the *default* notation, which is invoked if the semantic macro is used without an optional argument – hence, `$\eq{a}{b}$` still yields $a = b$.

If we use the starred variant of the `\notation` macro, the notation is set as the new default. Hence, had we done

```
\notation*{eq}[equiv]{#1 \mathrel{\maincomp{\equiv}} #2}
```

then `$\eq{a}{b}$` would now yield $a \equiv b$.

Any already existing notation can be set as default using the `\setnotation` macro; e.g. instead of using `\notation*`, we could also do

```
\notation{eq}[equiv]{#1 \mathrel{\maincomp{\equiv}} #2}
\setnotation{eq}{equiv}
```

Exercise

Implement the symbol "equal" as above in a new module "`Equality`" and add a documentation such that hovering over the symbol in the HTML yields the following snippet:

> Two objects a, b are considered **equal** (written $a = b$ or $a \equiv b$), if there is no property that distinguishes them.

Solution: Can be found in `[sTeX/MathTutorial]/mod/Equality1.en.tex`

8.2 Types and Variables

You might have noticed – after you save the file – that the expressions `$\eq{a}{b}$` and `$\eq[equiv]{a}{b}$` are underlined in yellow in the IDE and have a warning attached to them (Figure 8.1). If we click on the **`Invalid Unit`** link in the error message, we get a somewhat cryptic stacktrace-like window (Figure 8.2). The reason being, that MMT actually tries to formally verify *everything we write using semantic macros!* It does so,

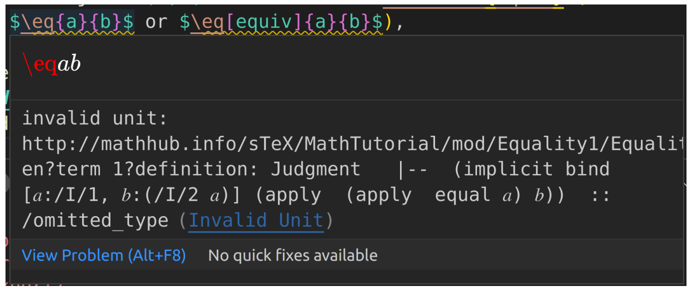

Figure 8.1: Type Checking Warning

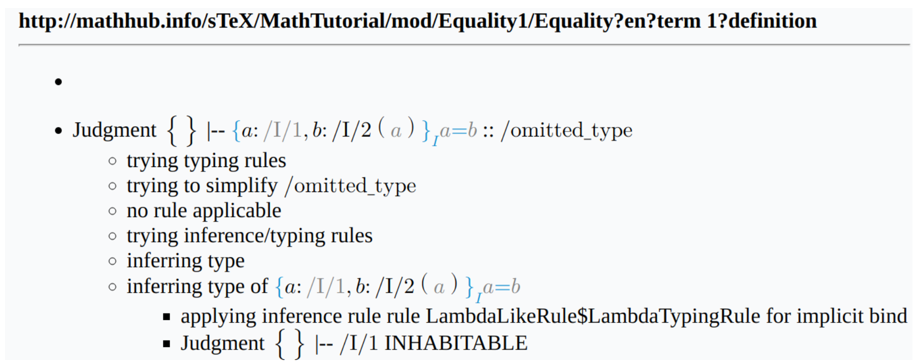

Figure 8.2: Type Checking Proof Tree

by attempting to infer the *type* of an expression – success implies that the expression is in fact well-typed.

If the former paragraph is difficult to comprehend for you, don't worry – you'll likely pick up on things as we go along. For now, sufice it to say that we can assign "*types*" to symbols, and the MMT system is smart enough to use those to check that what we're writing actually "makes sense"; for example, $a + b$ makes perfect sense if $+$ is addition and a and b are numbers, or elements of a vector space, but not if a and b are, say, triangles.

sTeX Every symbol or variable can be assigned a **type**, signifying what "kind of object" the symbol represents, or what (primary) set it is contained in.

In order to *formally verify* a mathematical statement, we have to rely on a set of *rules* that determine what is or isn't a valid statement. There are many systems

of such rules with very different flavours, called **(logical) foundations**.
The most commonly used foundation in (informal) mathematics is *set theory*, in particular *ZFC*; a set of axioms in (usually) *first-order logic*. However, in *computer proof assistants* and similar systems, *type theories* like *higher-order logic* or the *calculus of (inductive) constructions* are more popular, because they lend themselves better to computer implementations.
In as far as possible, we prefer to remain "foundationally agnostic", or **foundation independent**: Every foundation has advantages and disadvantages, and which one is appropriate often depends on the particular setting one is working in.
Nevertheless, certain "meta-principles" have proven themselves to be extremely effective in representing and checking mathematical content in software, and while we do not fix a particular foundation or specific checking rules, we will make use of those principles in general. These include e.g. the *Curry-Howard Correspondance*, or *Judgments-as-Types paradigm*, and *Higher-Order Abstract Syntax*.

Full formal verification of document content is an extremely lofty goal, and hardly realistic if you're not willing to write your content in pretty specific ways, and informed by a decent amount of background knowledge in formal logic. Moreover, formally verifying content in sTeX is an ongoing research project, so we will not go into the specifics in detail here.

While full formal verification is out of reach for now, annotating adequate types can strike a useful balance between the effort required and the benefit of automated meaning checking afforded by them. In this sense sTeX is pragmatically similar to programming languages where adding types can raise the quality and correctness assurance in programs.

Keep in mind that getting `Invalid Unit` warnings does not impact at all what your document is going to look like – feel free to ignore them entirely.

Types are particularly useful for *variables*:

A **variable** represents a *generic* or *unspecified* object.
Variables can be declared using the `\vardef`-macro, whose syntax is analogous to `\symdef`.
Note that variables are local to the current TeX-group (e.g. environment).

Let's leave our equality-module aside for now and turn our attention to something simpler: natural numbers. Consider the following module:

Example 5
Input:

```
\begin{smodule}{Nat}
  \symdef{Nat}[name=natural numbers]{\mathbb N}
  \begin{sparagraph}[style=symdoc]
    The \definame{Nat} $\defnotation{\Nat}$ are the numbers
    $0,1,2,...$
  \end{sparagraph}
  \symdef{plus}[name=addition,args=2]{#1 \mathbin{\maincomp{+}} #2}
  \begin{sparagraph}[style=symdoc]
    \Definame{addition} $\defnotation{\plus{a}{b}}$
    refers to the process of adding two \sn{Nat}.
  \end{sparagraph}
\end{smodule}
```

Output:

The **natural numbers** $\mathbb{N}$ are the numbers 0,1,2,...
Addition $a+b$ refers to the process of adding two natural numbers.

(like `\definame` and `\definiendum`, the `\defnotation` macro is only allowed in documenting environments like `sparagraph[style=symdoc]` or `sdefinition`, and highlights the notation components marked with `\comp` or `\maincomp` the same way as `\definame` and `\definiendum` do.)

Note, that as the `\Nat` semantic macro does not take any arguments, we do not need to wrap the notation in a `\comp` or `\maincomp`.

Note also, that the `\plus{a}{b}` is again underlined in the IDE with an `Invalid Unit` warning.

The above fragment uses two variables a and b. In fact, MMT will consider them variables even though they are not marked up as such – but since they are not marked up, we are missing out on useful functionality.

Let's change that by adding two variable definitions[1]:

Example 6

Input:

```
  \begin{sparagraph}[style=symdoc]
    \vardef{va}[name=a]{a}\vardef{vb}[name=b]{b}
    \Definame{addition} $\defnotation{\plus{\va}{\vb}}$
    refers to the process of adding two \sn{Nat}.
  \end{sparagraph}
```

Output:

Addition $a+b$ refers to the process of adding two natural numbers.

[1] Technically, this is called a *variable reservation*, for those in the know.

Okay, so now a and b are gray, but besides that, we haven't achieved much yet. Let's change that by giving the variables the type ℕ:

Example 7

Input:

```
\begin{sparagraph}[style=symdoc]
  \vardef{va}[name=a,type=\Nat]{a}\vardef{vb}[name=b,type=\Nat]{b}
  \Definame{addition} $\defnotation{\plus{\va}{\vb}}$
  refers to the process of adding two \sn{Nat}.
\end{sparagraph}
```

Output:

Addition $a+b$ refers to the process of adding two natural numbers.

Now if we hover over the a and b (in the HTML), it will show us that their type is ℕ!

We can of course also assign types to symbols. In the IDE, find the symbol *"function space"* with semantic macro `\funspace` (in `[sTeX/MathBase/Functions]{mod?Function}`). The OMDoc preview window shows you how to use this symbol (Figure 8.3). This tells

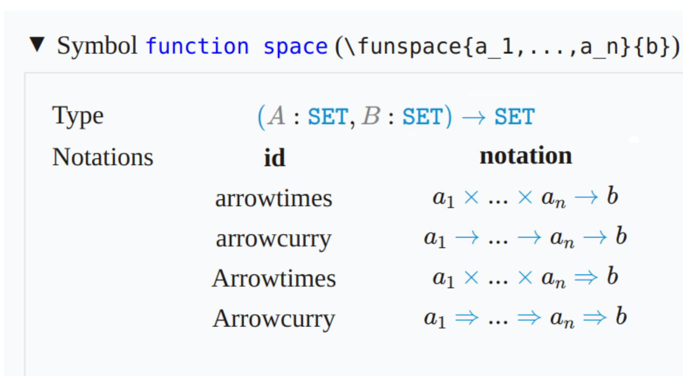

Figure 8.3: Syntax Preview

us that if we write `\funspace{a_1,...,a_n}{b}` (depending on which notation we use), we will get $a_1 \times \ldots \times a_n \to b$.

We want addition to have type $\mathbb{N} \times \mathbb{N} \to \mathbb{N}$, hence we do:

```
\symdef{plus}[name=addition,args=2,
  type=\funspace{\Nat,\Nat}{\Nat}
]{#1 \mathbin{\maincomp{+}} #2}
```

So far (and when using the **use** button in the IDE), we have been using the `\usemodule` macro to import content. `\usemodule` is allowed anywhere and imports the referenced module content local to the current TEX group.
Now that we use imported symbols in types (and since we are *in* a module), we need to make sure that the imported modules are also (transitively) *exported*, since our new symbols now *depend* on the imported module.
For that we use the `\importmodule` macro within the module; i.e. the file should now look something like this:

```
\begin{smodule}{Nat}
  \importmodule[sTeX/MathBase/Functions]{mod?Function}
  ...
```

Note that the HTML is aware of this now (after you save): *Clicking* on any occurence of addition now yields Figure 8.4.

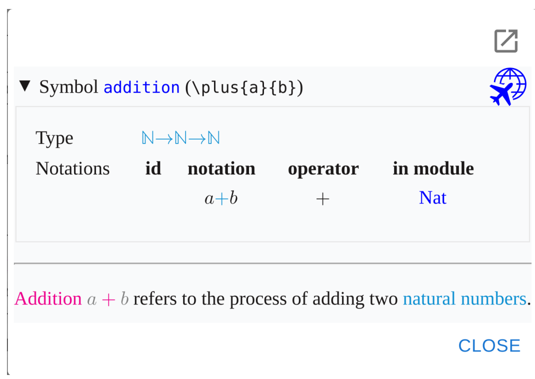

Figure 8.4: On-Click Popup in the HTML

However, the squiggly yellow `Invalid Unit` warnings are still there – that's because everything we did with types so far still depends on our natural numbers symbol, which does not have a type yet.

By virtue of using `[sTeX/MathBase/Functions]{mod?Function}`, we also imported `[sTeX/MathBase/Sets]{mod?Set}`, which gives us the *"collection"* symbol. Let's use this as a type for the natural numbers:

```
\symdef{Nat}[name=natural numbers,type=\collection]{\mathbb N}
```

Now if we save the file, all the squiggly lines are gone. Moreover, if you look at the OMDoc tab in the preview window, you can find Figure 8.5. The **Document Elements** block collects all semantically annotated expressions in a module or document; including variables and the `$\plus{\va}{\vb}$`. Here, it tells us that it has checked the expression $a + b$ (in the context of $a : \mathbb{N}$ and $b : \mathbb{N}$), and inferred that it has type $\mathbb{N}$.

Here's what just happened:

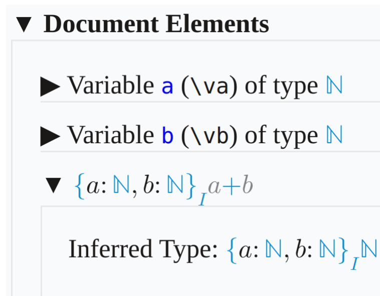

Figure 8.5: Inferred Type

1. The MMT system realized, that `$\plus{\va}{\vb}$` is the symbol "addition" applied to the two arguments a and b.
2. It knows, that "addition" has type $\mathbb{N} \times \mathbb{N} \to \mathbb{N}$[2].
3. It knows, that this means that if the two arguments a and b both have type $\mathbb{N}$, then the full expression has type $\mathbb{N}$.

Here's something you can now try: If we *remove* the types from the variables a and b again, the warnings are *still* gone. We lose the type information on hover, but MMT still doesn't complain, because it now realizes that since a and b have no explicit types given, it should infer them. And by the same chain of reasoning as above, it can infer that since they are being used as arguments for addition, they need to have type $\mathbb{N}$.

8.3 Flexary Macros and Argument Modes

Here is one thing you might wonder: Writing `$\plus{a}{b}$` is one thing, but what if we want to produce $a + b + c + d + e$? Do we really need to write `$\plus{a}{\plus{b}{\plus{c}{...}}}$`?

Of course not. We can declare the symbol such that the semantic macro `\plus` expects a (comma-separated) *sequence* of arguments instead of two "normal" arguments.

sTeX

The optional `args`-argument of `\symdecl` expects a string of characters indicating the semantic macro's **argument modes**. There are four such modes:

`i` a **simple argument**,

`a` a – *(left or right) associative* – **sequence argument**, represented as a single TeX-argument `{a,b,...}`,

[2] Do not worry that the IDE actually reports the type $\{a : \mathbb{N}, b : \mathbb{N}\}_I \mathbb{N}$, this is an artefact of the underlying type system with dependent types used by sTeX; it just means $\mathbb{N} \times \mathbb{N} \to \mathbb{N}$ in this special case, but would also allow a and b to appear in the range type in more complex situations; see ?? for details.

sTeX

- b A **binding argument** that expects a variable that is bound by the symbol in its application, and
- B A **binding sequence argument** of arbitrarily many bound variables by the symbol (`{x,y,z,...}`).

If `args` is given as a number n instead, the semantic macro takes n arguments of mode `i`.

Example 8

- For `\plus{a,b,c}` yielding $a + b + c$, we do `\symdecl{plus}[args=a]`,
- for `\inset{a,b,c}{A}` yielding $a, b, c \in A$, we do `\symdecl{inset}[args=ai]`,
- in `\add{i}{1}{n}{f(i)}` yielding $\sum_{i=1}^{n} f(i)$, the variable i is bound in the expression, we hence do `\symdecl{add}[args=biii]`,
- in `\foral{x,y,z}{P(x,y,z)}` yielding $\forall x, y, z.\ P(x, y, z)$, the variables x, y, z are all bound by the $\forall$, we hence do `\symdecl{foral}[args=Bii]`.

So when we wrote `\symdecl{plus}[args=2]`, this was actually shorthand for `\symdecl{plus}[args=ii]`.

Let's revise our previous declaration and the syntax of the `\plus` macro:

```
\symdef{plus}[name=addition,args=a,
  type=\funspace{\Nat,\Nat}{\Nat}
]{#1 \mathbin{\maincomp{+}} #2}
\begin{sparagraph}[style=symdoc]
  \vardef{va}[name=a]{a}\vardef{vb}[name=b]{b}
  \Definame{addition} $\defnotation{\plus{\va,\vb}}$
  refers to the process of adding two \sn{Nat}.
\end{sparagraph}
```

Now we get new errors, that are easy to explain: Our notation `{#1 \mathbin{\maincomp{+}} #2}` refers to *two* arguments, but our semantic macro only takes *one* (albeit a sequence argument). We now need to let sTeX know what to do with the sequence argument in our notation. Using the `\argsep` macro, we can tell sTeX to insert the *separator* "+" between the individual elements of the argument sequence `#1`:

```
\symdef{plus}[name=addition,args=a,
  type=\funspace{\Nat,\Nat}{\Nat}
]{\argsep{#1}{\mathbin{\maincomp{+}}}}
```

Now we can finally write `$\plus{a,b,c,d,e}$` and get $a + b + c + d + e$ – hooray!

...expect that our squiggly yellow `Invalid Unit` warnings are back. That's because the type of addition still corresponds to a binary operation, rather than a unary function on sequences.

We *could* change the type of course, but we shouldn't *want* to or *have* to: platonically, addition is *still* a *binary function*; we just introduced the `a`-mode argument for *our* convenience as authors.

Instead, we can tell MMT how to "resolve" the sequence argument into a nested application of addition. In the very common case we have here, where the symbol represents an *associative binary operator*, we can just add the argument `assoc=bin` to the `\symdecl` (or `\symdef`) macro:

```
\symdef{plus}[name=addition,args=a,assoc=bin,
  type=\funspace{\Nat,\Nat}{\Nat}
]{\argsep{#1}{\mathbin{\maincomp{+}}}}
```

and the warnings are gone again. Formally/internally, MMT will now turn the term `addition(sequence(a,b,c))` into `addition(a,addition(b,c))`.

Exercise

Analogously to the above, implement a symbol "`multiplication`" with semantic macro `\mult`, that takes a single sequence argument and has a default notation such that `\mult{a,b,c}` produces $a \cdot b \cdot c$.

Solution: Can be found in `[sTeX/MathTutorial]mod/Nat.en.tex`

8.4 Precedences

If you have done the previous exercise, you now have semantic macros `\plus` and `\mult` at your disposal. We can of course nest them to produce e.g. $a + b \cdot c$ (with `$\plus{a,\mult{b,c}}$`). If we do `$\mult{a,\plus{b,c}}$` however, we get $a{\cdot}b{+}c$. Annoying – we now have to insert parentheses: `$\mult{a,(\plus{b,c})}$`... or do we?

We do *not*. Instead, we can assign *precedences* to notations to have sTeX insert parentheses automatically.

sTeX

`\notation` (and hence `\symdef`) take an optional argument `prec=<opprec>;<argprec1>x...x<argprec n>` consisting of an **operator precedence** `<opprec>` and for each argument `k` an **argument precedence** `<argprec k>`.

All precedences are integers, e.g. 10 or -500. It is good practice to use precedences that leave enough room to smuggle values inbetween, so that we can fine-tune them later as more symbols may intervene.

The precise numbers used for precedences are arbitrary – what matters is which precedence is higher than which other precedence when used together.

By default, all precedences are 0, unless the symbol takes no arguments, in which case the operator precedence is `\neginfprec` (negative infinity).

If we only provide a single number in `prec=`, this is taken as both the operator precedence and all argument precedences.

The *lower* a precedence, the *stronger* a notation binds its arguments. In our particular case, we want multiplication to bind stronger than addition, so we can (arbitrarily) assign them precedences e.g. 10 and 20:

```
\symdef{plus}[name=addition,args=a,assoc=bin,prec=20,
  type=\funspace{\Nat,\Nat}{\Nat}
]{\argsep{#1}{\mathbin{\maincomp{+}}}}
\symdef{mult}[name=multiplication,args=a,assoc=bin,prec=10,
  type=\funspace{\Nat,\Nat}{\Nat}
]{\argsep{#1}{\mathbin{\maincomp{\cdot}}}}
```

And now if we type `$\mult{a,\plus{b,c}}$`, sTeX will automatically insert parentheses and yield $a \cdot (b + c)$ – and conversely, if we do `$\plus{a,\mult{b,c}}$`, sTeX will *not* insert parentheses and yield $a + b \cdot c$.

8.5 Implicit Arguments

Let us turn our attention back to equality. Here's an almost philosophical question: *What is the type of "equality"*? Asking (the right kind of) mathematicians this question can cause fist fights to break out. As such, we will not give a definitive answer, *but* here is an informative approach that has proven to be quite effective in computational settings:

Equality is a *polymorphic binary relation* on an *implicit* collection A. And a *relation* is a function into a type of *propositions*.

We will see the advantage of this approach over time. For now, consider that given objects a and b, the expression "$a = b$" is either true or false[3], and "equal" takes two arguments, so if we have a type of "truth values", it makes sense to model "equal" as a function taking two arguments and returning that type. So we do `type=\funspace{......`?

Here's the idea with respect to *implicit arguments*. Let's first declare a new variable of type "collection":

```
\vardef{vA}[name=a,type=\collection]{A}
```

We now assign the type $A \times A \to$ `Prop` to equal:

```
\symdef{eq}[name=equal,args=2,eq,
  type=\funspace{\vA,\vA}{\prop}
]{#1 \mathrel{\maincomp{=}} #2}
```

(The symbol "proposition" with semantic macro `\prop` comes with sTeX directly; we say that it is part of the sTeX.)

Now our type has a free variable A. For Mmt, this now means that equal actually takes *one more argument*, but one whose value is uniquely determined from the other arguments. Indeed, if you consider equal to take three arguments (the first one being some A of type collection), then the *next* two arguments *enforce* that the first argument has to be the type of the other two.

[3] Assuming classical logic – if you prefer to remain intuitionistic/constructive, note that sTeX, being foundation independent, does not enforce the law of excluded middle!

In other words: A is now an implicit argument that MMT is tasked with inferring whenever we use equal, and that we never explicitly provide in sTEX.

Indeed, if we use our module Nat from before, and apply \eq to a variable of type $\mathbb{N}$, MMT does not complain:

```
\usemodule{mod?Nat}
\vardef{vn}[name=n,type=\Nat]{n}
$\eq{\vn}{m}$
```

And if we inspect the OMDoc tab in the HTML preview, we can see exactly what MMT did (Figure 8.6). We can see

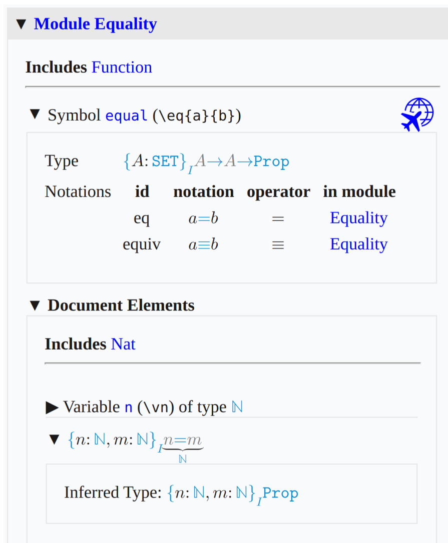

Figure 8.6: Implicit Arguments

1. (by the $\{\cdot\}_I \ldots$) that MMT considers A an implicit argument in the type of equal,
2. that the *inferred* type of $n = m$ is Prop,
3. that MMT inferred the implicit argument of equal in $n = m$ to be $\mathbb{N}$ (by the $\underbrace{\ldots}_{\mathbb{N}}$), and
4. that it was enough to give \vn the explicit type $\mathbb{N}$ – MMT also inferred that hence m also has to have type $\mathbb{N}$!

8.6 Finishing Equality

You might wonder if – as with addition – we can make "equal" take a sequence argument as well. Naturally, we can:

```
\symdef{eq}[name=equal,args=a,eq,
  type=\funspace{\vA,\vA}{\prop}
]{\argsep{#1}{\mathrel{\maincomp=}}}
\notation{eq}[equiv]{\argsep{#1}{\mathrel{\maincomp\equiv}}}
```

and as before, we now get `Invalid Unit` warnings. Unlike before, however, we can not just fix this with adding `assoc=bin`. As mentioned, `bin` instructs MMT to "fold" the symbol over the arguments, so when doing `\eq{a,b,c}`, MMT would turn this into `equal(a,equal(b,c))`, i.e. the claim that *"a" is equal to "$b = c$"* – but that's not what $a = b = c$ means. What we mean by $a = b = c$ is really "$a = b$ *and* $b = c$".

For that, we can use `assoc=conj` – however, that requires that some symbol that can be used for *conjunction* (i.e. "and") is in the current scope.

If we search for `conjunction` in the IDE, we should find the module `[sTeX/Logic/General]{mod/syntax?Conjunction}`.

Using that, we can now write the following:

```
\usemodule{mod?Nat}
\usemodule[sTeX/Logic/General]{mod/syntax?Conjunction}
\vardef{vn}[name=n,type=\Nat]{n}
$\eq{\vn,m,p}$
```

Upon saving, MMT does not complain; and if we inspect the OMDoc tab in the HTML window again, we now notice that MMT correctly resolved this as in Figure 8.7.

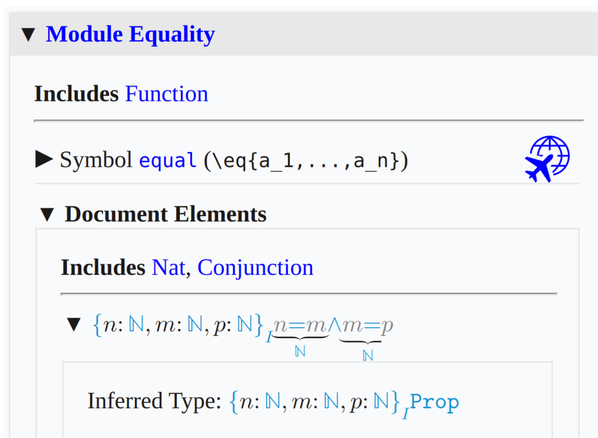

Figure 8.7: Conjunction of Equalities

8.7 Variable Sequences

There is a special kind of variable in sTeX for when we want to use *sequences* of variables.

We can use the `\varseq` macro to declare a new sequence variable; in the simplest case that looks something like the following:

```
\varseq{seqn}[name=n,type=\Nat]{1,\ellipses,k}{\maincomp{n}_{#1}}
```

We have just declared a new variable sequence of type $\mathbb{N}$, that ranges over indices $1, \ldots, k$, with notation n_i for some specific index i.

If we now do `\seqn{i}`, we get n_i, and if we do `\seqn!`, we get $n_1, \ldots, n_k$.

We can also do multi-dimensional sequences, e.g.

```
\varseq{seqm}[name=m,type=\Nat,args=2]
  {{1}{1},\ellipses,{\ell}{k}}
  {\maincomp{m}_{#1}^{#2}}
```

Now `\seqm{i}{j}` produces m_i^j, and `\seqm!` produces $m_1^1, \ldots, m_\ell^k$.

Of course, we can manually change the way `\seqn!` is typeset by providing an explicit operator notation using `op=`; e.g. if we do

```
\varseq{seqn}[name=n,type=\Nat,op={(n_i)_{i=1}^k}]
  {1,\ellipses,k}{\maincomp{n}_{#1}}
```

then `\seqn!` produces $(n_i)_{i=1}^k$.

So far so nice, but sequence variables get especially useful in combination with sequence arguments: Consider for example the `\plus` semantic macro for addition. This expects one sequence argument, or alternatively, a *sequence variable*: `\plus{\seqn}` now produces $n_1 + \ldots + n_k$, and `\eq{\seqm}` now produces $m_1^1 = \ldots = m_\ell^k$.

TODO[4]

[4] TODO: seqmap

Chapter 9

Statements

Now that we have equality, natural numbers, addition and multiplication at our disposal, let's implement some *statements*. Both addition and multiplication are, for example, *associative* and *commutative*.

We could state these properties directly for the two operations, but we can also first define *associativity* and *commutativity* in general, and then assert them specifically for addition and multiplication.

9.1 Definitions

Let's define what it means to be *associative*. This means, of course, declaring a new symbol. Note that we don't need a semantic macro for associativity, since there is no notation to attach to it. We will also for now ignore its type. Note however, that associativity is still a property of (binary) operations, so it still makes sense to have the symbol take an *argument*; namely the operation it applies to.

We will also finally provide an actual (more or less) formal *definition* for the symbol, so where we used the `sparagraph` environment with `style=symdoc` before, we will now use the `sdefinition` environment, which also gives us `\definame`, `\definiendum`, `\defnotation` and all that.

A first variant of a corresponding module could look like this:

Example 9

Input:

File [sTeX/MathTutorial]**props/Associative1.en.tex**

```
\begin{smodule}{Associative}
  \importmodule{mod?Equality}

  \symdecl*{associative}[args=1]
  \begin{sdefinition}[for=associative]
    \vardef{vA}[name=A,type=\collection]{A}
    \vardef{vop}[name=op,type=\funspace{\vA,\vA}\vA,args=a,assoc=bin]
      {\argsep{#1}{\mathbin{\maincomp{\circ}}}}
    %
    A binary operation $\fun{\vop!}{\vA,\vA}\vA$ is called
    \definame{associative}, if
    $\eq{
      \vop{(\vop{a,b}),c},
      \vop{a,(\vop{b,c})}
    }$ for all $\inset{a,b,c}\vA$.
  \end{sdefinition}
\end{smodule}
```

Output:

Definition 9.1.1. A binary operation $\circ : A \times A \to A$ is called **associative**, if $(a \circ b) \circ c = a \circ (b \circ c)$ for all $a, b, c \in A$.

Note, that the semantic macros `\fun` and `\inset` come from `[sTeX/MathBase/Functions]mod?Function` and `[sTeX/MathBase/Sets]mod?Set`, respectively. Also note, that the variable declaration for `\vop` makes use of all the fun features we already discussed for addition.

Note that the above is more than good enough, if you merely want to produce nice-looking, "wikified" HTML and PDF documents. The rest of this section will cover how to add more flexiformal semantics to the above.
If this seems laborious and/or difficult, keep in mind that this is to some degree experimental still, and you are not forced to go overboard with semantic annotations!
But if you aim to create a "library of symbols" for mathematical concepts, then all of the possibilities that we discuss here will add value for the community. Generally, the higher the ratio of readers to authors the more any investment in semantization will pay off.

9.1.1 Semantic Macros in Text Mode

The first thing we can do to further improve this document is marking up the "for all" in the definition – after all, there naturally is a symbol for the universal quantifier, which can be found in `[sTeX/Logic/General]mod/syntax?UniversalQuantifier` and has the semantic macro `\foral` (as to not conflict with the TeX primitive macro `\forall`).

The naive approach would be to replace the "for all" by e.g. `\sr{foral}{for all}`. That would (correctly) associate and highlight the text fragment with the symbol "universal quantifier", *but* we are not just referencing the symbol here – we are actually using it, by *applying* it to the variables a, b, c and the expression $(a \circ b) \circ c = a \circ (b \circ c)$.

In *math mode*, we can just use the semantic macro `\foral` – that will take two arguments (of modes `B` and `i`) and produce the corresponding notation, so that

```
$\foral{\inset{a,b,c}{\vA}}{
  \eq{ \vop{(\vop{a,b}),c} , \vop{a,(\vop{b,c})} }
}$
```

will produce $\forall a, b, c \in A.(a \circ b) \circ c = a \circ (b \circ c)$.

In *text mode*, however, we don't have a specific notation – instead, the specific "notation" is whatever sentence we want to mark up semantically. In text mode, semantic macros therefore behave differently:

1. They take *precisely* one argument, regardless of how many arguments the macro would take in math mode or (equivalently) the `args` property of the symbol.

2. *Within* that argument, we can use `\comp` to highlight arbitrary text fragments, and

3. we can use the `\arg` macro to mark up the *actual* arguments that the symbol is supposed to be applied to.

   `\arg` takes as optional argument the index of the argument that is being marked up; if not they are used consecutively. The starred variant `\arg*` produces no output.

So we could now do

```
\foral{\comp{For all} $\arg{\inset{a,b,c}{\vA}}$, we have
  $\arg{
    \eq{ \vop{(\vop{a,b}),c} , \vop{a,(\vop{b,c})}}
  }$
}
```

which produces "For all $a, b, c \in A$, we have $(a \circ b) \circ c = a \circ (b \circ c)$".

In our case though, we want to "switch the arguments around" – first comes the equation, then the variables to be bound. Hence:

```
\foral{
  $\arg[2]{
    \eq{ \vop{(\vop{a,b}),c}, \vop{a,(\vop{b,c})} }
  }$
  \comp{for all}
  $\arg[1]{ \inset{a,b,c}{\vA} }$
}
```

which produces "$(a \circ b) \circ c = a \circ (b \circ c)$ for all $a, b, c \in A$".

9.1.2 Definientia

Now we have a fully semantically annotated expression in the definition for “associative”. Can we let MMT know, that this expression really is *the* definition of the symbol?

Yes, we can. All we need to do is wrap the sentence in a `\definiens` macro (plural: *definientia*; like the word *“definiendum”* refers to “the term being defined”, *“definiens”* refers to “the thing the term is being defined *as*”).

The `\definiens` macro is only allowed within the `sdefinition` environment, and requires that the environment lists the symbol that gets the definiens attached explicitly in its `for=` argument. It is possible to attach definientia to multiple symbols within an `sdefinition` environment, in which case the symbol needs to be provided as an optional argument, e.g. we could do `\definiens[associative]{...}`. Since “associative” is the only symbol being defined in our definition, we can omit that argument.

Alternatively, as with types we can attach definientia to a `\symdecl` directly using the optional argument `def=...`.

At this point, you might justifiably wonder, why we even still need to declare associative with `\symdecl*` before we define it. And indeed, we don’t – the `sdefinition` environment takes the same optional arguments as the `\symdecl` macro, and if we explicitly provide a `name=` (or a `macro=`), it will generate a symbol for us. We can hence get rid of the `\symdecl*` and instead do:

```
\begin{sdefinition}[name=associative,args=1]
  ...
\end{sdefinition}
```

One more problem remains: We stated that associative is to take one argument – but we haven’t told sTeX what it is yet. In our case, the argument is represented by the variable `\vop`. In fact, chances are that arguments to symbols in types or definientia are almost always represented by some variable.

We can use one of two ways to a variable as being an argument:

1. If the variable (e.g. `\vop` with name `op`) was already declared prior to the `sdefinition` environment, we can use the `\varbind` macro in the environment; e.g. by adding `\varbind{op}`.

2. We can move (or copy) the `\vardef` for the variable into the environment and add `bind` to its optional arguments.

In total, our fully annotated definition now looks like this:

Example 10
Input:

File [sTeX/MathTutorial]props/Associative.en.tex

```
\begin{sdefinition}[name=associative,args=1]
  \vardef{vA}[name=A,type=\collection]{A}
  \vardef{vop}[name=op,type=\funspace{\vA,\vA}\vA,
    args=a,assoc=bin,bind % <- argument for the symbol
  ]{\argsep{#1}{\mathbin{\maincomp{\circ}}}}
  \vardef{va}[name=a,type=\vA]{a}
  \vardef{vb}[name=b,type=\vA]{b}
  \vardef{vc}[name=c,type=\vA]{c}
  %
  A binary operation $\fun{\vop!}{\vA,\vA}\vA$ is called
  \definame{associative}, if
  \definiens{\foral{$\arg[2]{\eq{
    \vop{(\vop{\va,\vb}),\vc},
    \vop{\va,(\vop{\vb,\vc})}
  }}$ \comp{for all} $\arg[1]{\inset{\va,\vb,\vc}\vA}$}}.
\end{sdefinition}
%
```

Output:

Definition 9.1.2. A binary operation $\circ : A \times A \to A$ is called **associative**, if $(a \circ b) \circ c = a \circ (b \circ c)$ for all $a, b, c \in A$.

And indeed, if we look at the OMDoc tab of the HTML preview, we can see that not only does Mmt attach the definiens to the symbol, it has also inferred the type of "associative" from the definiens (Figure 9.1).

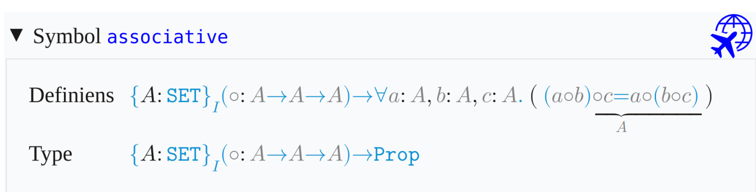

Figure 9.1: Type Inferred from Definiens

9.1.3 Using Symbols Without Semantic Macros and Exporting Code in Modules

So now we don't have a semantic macro for "associative", but it *does* take an argument. How can we ever actually *use* the symbol now?

The answer is: with the `\symuse` macro. Like `\symref` and friends, `\symuse` takes a symbol name or the name of its semantic macro as argument, but behaves otherwise like using a semantic macro directly. So for, say, addition, \symuse{*addition*} and \symuse{*plus*} behave exactly like \plus.

In our case, this means we can do \symuse{*associative*}. "associative" does not have a notation, but in practice, we say something like *"+ is associative"* rather than using some specific mathematical notation for the same thing.

Combining this with what we just learned, we can now say that addition is associative by doing:

```
\symuse{associative}{$\arg{\plus!}$ \comp{is associative}}
```

In fact, we would do the exact same thing every time we want to say that *some* operator is associative, so it makes sense to introduce a macro for this. In fact, such a macro is easy to define using standard LaTeX methods. This is where \STEXexport becomes very handy:

In a module, we can put arbitrary LaTeX code in an \STEXexport, and this code will be executed every time the module is imported via \usemodule or \importmodule. This is especially useful for macro definitions, and this way modules can almost act like LaTeX packages!

So we can define a new macro \isassociative that applies "associative" to an arbitrary operation and produces the semantically marked-up text "#1 is associative", and wrap that macro definition in an \STEXexport, and whenever we use the Associative module, we also get the \isassociative macro:

```
\STEXexport{
  \def\isassociative#1{
    \symuse{associative}{\arg{#1} ~is ~\comp{associative}}
  }
}
```

And now, we can do e.g. \isassociative{$\plus!$} to produce "+ is associative".

For technical reasons, \STEXexport processes its content in the expl3 category code scheme – what this means is that all spaces are ignored entirely, and the characters _ and : are valid characters in macro names.
In practice, this means you will have to use the ~ character for spaces, and if you want to use a subscript _, you should use the macro \c_math_subscript_token instead.

Exercise
Analogously to all the above, implement a module for *commutativity*; i.e the property of a binary operation that $a \circ b = b \circ a$ for all a, b. Make the module export a macro \iscommutative analogously to \isassociative.

Solution: Can be found in [sTeX/MathTutorial]props/Commutative.en.tex

TODO[1]

[1]TODO: intent?

9.2 Assertions

Having defined associativity and commutativity, we can now assert that both properties hold for addition and multiplication.

For *assertions* (i.e. theorems, lemmata, axioms, claims,...), sTEX provides the `sassertion` environment.

In the simplest case, that can look like the following:

```
\begin{sassertion}
  \isassociative{\Sn{plus}}
\end{sassertion}
```

which yields

Addition is associative

Do we want this to be typeset as a **Theorem**? For that we just add a `[style=theorem]` to the `sassertion` environment, provided we have a customization for that – (see chapter 9 (User Manual) in the sTEX Documentation). We can also load the `stexthm` package, which uses the `amsthm` package to provide common typesettings for the types: `theorem`, `observation`, `corollary`, `lemma`, `axiom` and `remark`.

So far, this is not too useful – after all, we could have just as well used e.g. the `amsthm` package and gone straight for the non-sTEX variant

```
\begin{theorem}
  \isassociative{\Sn{plus}}
\end{theorem}
```

But as with `sdefinition`, we can immediately add a corresponding symbol in the `sassertion` `environment`, and have it be documented directly by the environment:

```
\begin{sassertion}[style=theorem,name=addition is associative]
  \isassociative{\Sn{plus}}
\end{sassertion}
```

And now, if we do `\sn{addition is associative}`, we get addition is associative with a corresponding hover pop-up (in the HTML).

Of course, the usefulness of this grows with more elaborate assertions. For very short assertions such as the above, we might not even want to typeset them in such a space hungry manner.

For that purpose, we provide the `\inlineass` macro (and analogously: `\inlinedef` for `sdefinition`), which takes the same optional arguments as the environment. So we could also do:

```
\inlineass[name=addition is associative]{\isassociative{\Sn{plus}}}
```

So far, MMT is blissfully unaware of the semantic contents of our assertions. We can easily remedy that by wrapping the expression representing the assertion in a `\conclusion` macro, analogously to the definiens macro in `sdefinition`s:

```
\inlineass[name=addition is associative]{
  \conclusion{\isassociative{\Sn{plus}}}
}
```

We can now see the statement in the OMDoc tab of the HTML preview (Figure 9.2).

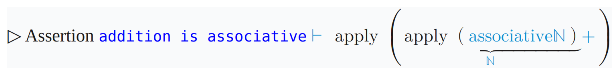

Figure 9.2: Assertion Statement in OMDoc

Not exactly pretty – the OMDoc tab uses notations to render content, and we did not provide any for associative.

Notice the ⊢ symbol after the name of the assertion? As an aside for those who are curious:

> sTeX
>
> The **judgments as types** paradigm represents the validity of proposition via a designated *type of proofs*: For any proposition P, we introduce a collection $\vdash P$ of *proofs* of P.
> To say that the proposition *holds* is then equivalent to positing that *some* element $p : \vdash P$ exists – in which case *proofs* become typed objects in their own right.

Let's consider a more interesting statement now. How about the induction axiom?

```
\begin{sassertion}[style=axiom,name=induction axiom]
  Let $\varphi(n)$ a property on \sn{Nat}. If
  \begin{enumerate}
    \item $\varphi(0)$ and
    \item if $\varphi(m)$ holds for some $m$, then
    $\varphi(\plus{m,1})$ also holds,
  \end{enumerate}
  then $\varphi(n)$ holds for all $\inset{n}{\Nat}$.
\end{sassertion}
```

Axiom 9.2.1. *Let $\varphi(n)$ a property on natural numbers. If*

1. *$\varphi(0)$ and*
2. *if $\varphi(m)$ holds for some m, then $\varphi(m+1)$ also holds,*

then $\varphi(n)$ holds for all $n \in \mathbb{N}$.

Exercise

Annotate the above by:

1. Variables with appropriate notations for φ, m and n, and

2. marking up the second premise ("if $\varphi(m)$ holds for some...") in text mode as the formula $\forall m.\varphi(m) \Rightarrow \varphi(m+1)$ using the semantic macros `\foral` (which we saw earlier already) and `\imply` (implication) from `[sTeX/Logic/General]mod/syntax?Implication`. The text fragments that should be highlighted are "`if`" and "`then`".

3. marking up the conclusion ("$\varphi(n)$ holds for all $n \in \mathbb{N}$") in text mode as the formula $\forall n.\varphi(n)$. The text fragment that should be highlighted is "`for all`".

Hint:

- The starred variant `\arg*{...}` produces no output.
- Giving a notation the precedence `prec=nobrackets` assigns precedences such that no parentheses are inserted around either the notation itself, or its arguments.
- `\dobrackets{...}` in a notation wraps its argument in parentheses, makes sure that no *additional* parentheses are automatically inserted in its argument, and highlights the parentheses themselves with `\comp`.
- So far, MMT does not know that 0 and 1 are natural numbers. While there are smarter (but more technical) ways to solve this, for now we recommend introducing symbols `zero` and `one` with notations 0 and 1, respectively.

Solution: Can be found in `[sTeX/MathTutorial]mod/NatTheorems.en.tex`

So how can we teach MMT the semantics of this statement? Here's what we can do:

1. As with the simpler assertions (and hence the name), the *conclusion* of the assertion can be marked up with `\conclusion`.
2. As with `sdefinition`, we can mark variables as *bound* (using either `bind` in the `\vardef` or `\varbind`). *If* a symbol that can act as a universal quantifier is in scope, variables marked as bound are abstracted away using that symbol.
3. Similarly to `\conclusion`, *premises* can be marked up as such using the `\premise` macro. If a symbol is in scope that can act as an implication, that will be used to connect the premise(s) to the conclusion.

Hence, if we mark the variable φ as bound and use `\premise` and `\conclusion` (see `[sTeX/MathTutorial]mod/NatTheorems.en.tex`), we can inspect the OMDoc tab in the HTML preview again and see that MMT has now constructed the proposition (Figure 9.3).

▷ Assertion `induction axiom` $\vdash \forall\varphi\colon\mathbb{N}\to\mathtt{Prop}.\varphi(0)\Rightarrow(\forall m\colon\mathbb{N}.\varphi(m)\Rightarrow\varphi(m+1))\Rightarrow(\forall n\colon\mathbb{N}.\varphi(n))$

Figure 9.3: The Induction Axiom in OMDoc

9.3 Proofs

sTeX provides the `sproof` environment for marking up *proofs*.
The markup mechanism for `sproof` is still highly experimental and likely subject to change in the near future. As such, we omit a closer explanation of its usage until the syntax and functionality have sufficiently stabilized.

Chapter 10

Mathematical Structures

A common concept in mathematics is that of a mathematical structure – a *tuple* of interdependent components. For example: A *monoid* is a structure $\langle M, \circ, e \rangle$ such that certain axioms hold; where M is a set, $\circ$ is a binary operation, and $e \in M$.

From a representational perspective, this is particularly interesting: M, $\circ$ and e in the above are not symbols in the same way that the previous symbols we considered were – they don't represent definite objects. Instead, they are *components* of some other object, namely a monoid; where a *particular* monoid could either be a fixed object (such as $\langle \mathbb{Z}, +, 0 \rangle$) or an *indefinite* monoid; i.e. a variable. We call the components of a mathematical structure **fields**.

In this chapter, we will discuss how to declare and use mathematical structures in sTeX, build them up modularly, and connect them among each other to avoid duplication.

We will do so by considering *lattices* both algebraically and order-theoretically, and identify the two perspectives.

10.1 Declaring and Using Structures

The simplest kinds of structures are *magmas* and *(directed) graphs*, so we might as well start there:

Definition 10.1.1. A **magma** is a structure $\langle U, \circ \rangle$, where U is a collection and $\circ$ a binary operation $U \times U \to U$.

The obvious start is to create a new module `Magma`. Within this module, we import the `Functions` module so we can later assign a type to the operation. We can then use the `mathstructure` environment, that creates a new symbol "magma":

```
\begin{smodule}{Magma}
  \importmodule[sTeX/MathBase/Functions]{mod?Function}
  \begin{mathstructure}{magma}
    ...
```

```
  \end{mathstructure}
\end{smodule}
```

`mathstructure` behaves very similarly as `smodule` – within the environment, we can declare new symbols, notations and all that.

So within the `mathstructure`, we can add symbols for the two fields U and $\circ$:

```
\symdef{univ}[name=universe,type=\collection]{U}
\symdef{op}[name=operation,args=a,assoc=bin,
  type=\funspace{\univ,\univ}\univ
]{\argsep{#1}{\mathbin{\maincomp{\circ}}}}
```

Once we close the environment (with `\end{mathstructure}`), the symbols are "gone". However, we now have a new symbol "magma" with semantic macro `\magma`. It's usage is somewhat more complicated than "normal" semantic macros, but one thing we *can* do with it now is `$\magma!$`, which will produce $\langle U, \circ\rangle$.

Notably however, the `\magma` macro is already available *within* the `mathstructure` environment as well.

This allows us to provide an `sdefinition` using the semantic macros declared in the structure:

Example 11

Input:

File [sTeX/MathTutorial]`algebra/Magma.en.tex`

```
\begin{mathstructure}{magma}
  \symdef{univ}[name=universe,type=\collection]{U}
  \symdef{op}[name=operation,args=a,assoc=bin,
    type=\funspace{\univ,\univ}\univ]
    {\argsep{#1}{\mathbin{\maincomp\circ}}}

  \begin{sdefinition}[for={magma,univ,op}]
    A \definame{magma} is a \sr{mathstruct}{structure} $\magma!$,
    where $\univ$ is a \sn{collection} and $\op!$
    a binary operation $\funspace{\univ,\univ}\univ$.
  \end{sdefinition}
\end{mathstructure}
```

Output:

Definition 10.1.2. A **magma** is a structure $\langle U, \circ\rangle$, where U is a collection and $\circ$ a binary operation $U \times U \to U$.

10.1.1 Instantiating Structures

More importantly however, we can now declare a variable magma, using the optional `return=` argument. For example, we can now do

```
\vardef{vM}[name=M,return=\magma]{M}
```

and we get the semantic macro `\vM` with which we can do the following:

Syntax	Result
`$\vM!$`	M
`$\vM{}$`	$\langle U_M, \circ_M \rangle$
`$\vM{univ}$`	U_M
`$\vM{op}!$`	$\circ_M$
`$\vM{op}{a,b,c}$`	$a \circ_M b \circ_M c$

In other words: Given a symbol or variable with semantic macro `\foo` and `return=\struct`, then `\foo{<fn>}` behaves like the semantic macro `\fn` *within* the `mathstructure` environment for `struct` – but instantiated for the specific instance `foo`.

By default, sTEX attaches the symbol's (or variable's) operator notation as a subscript suffix to the notation component marked with `\maincomp` – e.g., since the "`\circ`" in the notation for `op` is marked with `\maincomp`, doing `$\vM{op}{a,b}$` ultimately outputs `a \circ_{\vM!} b`. Hence, we get $a \circ_M b$.

We can change the way the `\maincomp` notation component is modified, by using the optional argument `copm=` in the semantic macro for the mathematical structure. For example, to not change it at all, we can do:

```
\vardef{vM}[name=M,return={\magma[comp=##1]}]{M}
```

...or to suffix it with a `'`, we can do

```
\vardef{vMp}[name=Mp,return={\magma[comp=##1']}]{M'}
```

This allows us to do things like:

```
Let $\vM!:=\vM{}$ and $\vMp!:=\vMp{}$ \sns{magma}. Then...
```

yielding

> Let $M := \langle U, \circ \rangle$ and $M' := \langle U', \circ' \rangle$ magmas. Then...

We can also *assign* fields to (arbitrary) expressions, by doing `name=<tex>` in square brackets. For example we can do the following:

```
\vardef{vA}[type=\collection]{A}
\vardef{vM}[name=M,return={\magma[comp=##1][univ=\vA]}]{M}
\vardef{vMp}[name=Mp,return={\magma[comp={{##1}'}][univ=\vA]}]{M'}

Let $\vM!:=\vM{}$ and $\vMp!:=\vMp{}$ \sns{magma} on $\vA$....
```

> Let $M := \langle A, \circ \rangle$ and $M' := \langle A, \circ' \rangle$ magmas.

Of course, we can also use `return=` with variable sequences – for example:

```
\varseq{vMs}[name=M,return={\magma[comp={##1}_{#1}]},op=(M_i)_1^n]
  {1,\ellipses,n}{\maincomp{M}_{#1}}
Let $\vMs! := \vMs{i}{}_1^n$ a sequence of \sns{magma}...
```

Let $(M_i)_1^n := \langle U_i, \circ_i \rangle_1^n$ a sequence of magmas...

Note that in the above, it seems that using `#1` in the `return` argument is allowed. Indeed, it is - the `return` statement takes the same arguments as the semantic macro itself does and is appropriately instantiated. Since the first (and only) argument to the sequence `\vMs` is the index, when doing `\vMs{i}...` the `#1` in the `return`-statement will be replaced by `i`.

Also, note that if we want to produce M_i – i.e. the magma at index i in the sequence, we can do `\vMs{i}!`.

Think of the `!` as a "stop sign" - if the expression up to the `!` has an associated presentation, the `!` tells sTEX to "stop eating arguments" and present whatever it has until now.

10.2 Extending Structures and Axioms

It is extremely common to "build up" structures in a hierarchical manner by adding new fields or axioms: A *semigroup* is an associative magma. A *band* is an idempotent semigroup. A *monoid* is a semigroup with a unit. A *partial order* is an antisymmetric preorder.

We alluded to the fact earlier, that the `mathstructure` environment behaves like an `smodule` – that is literally true: Every `mathstructure` `foo` in a module `FooMod` is in fact also a module `?FooMod/foo-module`. We can therefore easily extend structures using `\importmodule{...?FooMod/foo-module}` – but extending structures is so common, and using `\importmodule` tiring, that there is a shortcut: the `extstructure` environment. It takes as second argument a comma-separated list of structure names. That allows us to easily define semigroups:

Example 12

Input:

File [sTeX/MathTutorial]`algebra/Semigroup.en.tex`

```
\begin{extstructure}{semigroup}{magma}
  \begin{sdefinition}
    A \definame{semigroup} is a \sn{magma} $\semigroup!$,
    where \inlineass[name=associative axiom]{
      \conclusion{\isassociative{$\op!$}}.
    }
  \end{sdefinition}
\end{extstructure}
```

Output:

Definition 10.2.1. A **semigroup** is a magma $\langle U, \circ \rangle$, where $\circ$ is associative.

Note our usage of `\inlineass` to generate a new symbol for the associative axiom.

If we look at the OMDoc tab in the HTML preview window, we can see the output in Figure 10.1.

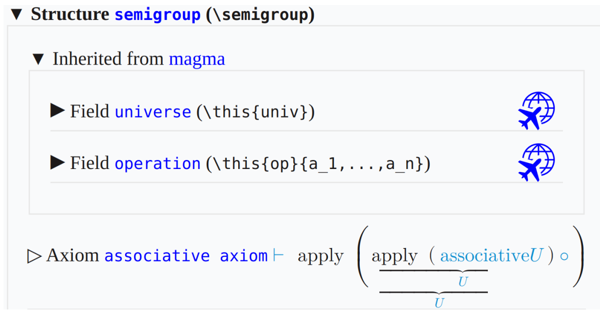

Figure 10.1: Axioms in OMDoc

So Mmt has decided that our statement is an *axiom*.

10.2.1 Conservative Extensions

For structures, there is a *critical* distinction between *defined* and *undefined* symbols; and analogously between *theorems* and *axioms*.

Remember that structures are more like *templates* that are *instantiated* by particular objects. An *undefined* field in a structure, in that sense, is like an *obligation*: If something is supposed to be a semigroup, it *has to* have a universe, an operation and the operation needs to satisfy the associative axiom.

Defined fields on the other hand have a *definiens* on the basis of the remaining fields – they don't need to be explicitly provided for something to instantiate the structure; if all the *undefined* fields are provided, the *defined* ones we get *"for free"*.

The same holds for *theorems*: If a statement is *provable* from the axioms, then we don't need to explicitly prove it to hold for some particular instance – we have a proof already, provided the axioms hold.

> The relation between axioms and theorems is not just analogous to that between undefined and defined symbols: It is the very same. Remember the judgments as types paradigm?
> For a proposition P, an assertion in sTeX induces a symbol of type $\vdash P$. Without a proof, this symbol is *undefined* – and hence an *axiom*. A *proof* for P is a specific term of type $\vdash P$ – i.e. a potential *definiens*. To prove an assertion turns it into a *theorem*, which is to say that the symbol can be *defined*.

One consequence of this is: Extending a structure only by *defined* fields does not actually (conceptually) introduce a *new* structure – every instance of the old one *should* also be an instance of the new one. The new fields are basically just "syntactic sugar".

There is a name for extending a structure only by defined fields (or theorems): A *conservative extension*.

sTeX provides the `extstructure*` environment for that purpose. Unlike `extstructure`, it does *not* take a name (technically, sTeX generates one internally). Instead, conceptually `extstructure*` modifies the extended structure directly, rather than generating a new structure. The caveat however is, that every symbol introduced in an `extstructure*` **must** be defined.

Consider the following conservative extension:

Example 13

Input:

File [sTeX/MathTutorial]`algebra/MagmaSquare.en.tex`

```
\begin{extstructure*}{magma}
  \begin{sdefinition}[macro=sq,args=1]
    \notation{sq}[op=\cdot^2]{{#1}^{\comp 2}}
    \vardef{va}[name=a,type=\univ,bind]{a}
    Let $\inset{\va}{\univ}$. We define
    $\defnotation{\sq{\va}} := \definiens{\op{\va,\va}}$.
  \end{sdefinition}
\end{extstructure*}
```

Output:

Definition 10.2.2. Let $a \in U$. We define $a^2 := a \circ a$.

Via `\definiens`, the new symbol sq is now *defined* (note the `macro=` argument, taht generates a semantic macro as well). Whenever we import the containing module, we now have an additional field `sq` in (any extension of) magma – e.g., the following is now valid:

```
\usemodule[sTeX/MathTutorial]{algebra?MagmaSquare}
\vardef{vsg}[name=S,return=\semigroup]{S}
$\vsg{sq}{a}$
```

...producing a^2.

10.3 Nesting Structures and `\this`

A prehaps not too surprising, but a notable aspect of structures is that fields themselves can be instances. This is important for example for implementing *vector spaces*, but can also be used to bundle things that are not normally thought of as structures, such as objects with certain defining properties.

Take as an example, the notion of a (magma) homomorphism:

Definition 10.3.1. Let $M_1 = \langle U_1, \circ_1 \rangle$ and $M_2 = \langle U_2, \circ_2 \rangle$ magmas. A **magma homomorphism** is a function $F : U_1 \to U_2$ such that $F(a \circ_1 b) = F(a) \circ_2 F(b)$ for all $a, b \in U_1$.

So a homomorphism is a function with certain properties. And structures can be used to "bundle" the function itself with both the magmas on whose universes the function operates, as well as the *axiom* that *makes* it a homomorphism. After all, considered as a mere function, $F : U_1 \to U_2$ contains no information about the operation with respect to which it is homomorphic.

The first thing to note is that we can provide `mathstructure` with an optional argument for a *name* distict from the name of its semantic macro. We then add two fields that `return` magmas. So far, so unexciting:

```
\begin{mathstructure}{magmahom}[magma homomorphism]
  \symdef{dom}[name=domain,return={\magma[comp={##1}_1]}]{M_1}
  \symdef{cod}[name=codomain,return={\magma[comp={##1}_2]}]{M_2}
```

For the function itself, we know how to give it a maningful type, already:

```
\symdef{f}[type=\funspace{\dom{univ}}{\cod{univ}},args=1]{???}
```

...but what should its notation be? Ideally we would want it to just be the notation of whatever particular instance it is – in informal mathematics, we rarely distinguish notationally between a homomorphism and its underlying function (to the point where it's not clear, whether *informally* the distinction is even meaningful). Similarly, we rarely distinguish e.g. between a magma (or semigroup, monoid, group, ring, vector space,...) and its underlying universe.

This is where `\this` comes into play (pun intended). Within an `mathstructure` or `exstructure`, or in the context of a particular instance of one, `\this` represents "the" instance.

We can set it in the context of `mathstructure` as a further optional argument; e.g.

```
\begin{mathstructure}{magmahom}[magma homomorphism,this=F]
```

and then use `\this` in the notation for the function. We can further provide the homomorphism condition as an axiom using `\inlineass`:

Example 14

Input:

File [sTeX/MathTutorial]algebra/Homomorphism.en.tex

```
\begin{mathstructure}{magmahom}[magma homomorphism,this=F]
  \symdef{dom}[name=domain,return={\magma[comp={##1}_1]}]{M_1}
  \symdef{cod}[name=codomain,return={\magma[comp={##1}_2]}]{M_2}
  \symdef{f}[op=\this,args=1,
    type=\funspace{\dom{univ}}{\cod{univ}}
  ]{\this \dobrackets{#1}}

  \begin{sdefinition}[for={magmahom,dom,cod,f}]
    \vardef{va}[name=a,type=\dom{univ}]{a}
    \vardef{vb}[name=b,type=\dom{univ}]{b}
    Let $\dom!=\dom{}$ and $\cod!=\cod{}$ \sns{magma}.
    A \definame{magmahom} is a function
    $\fun{\f!}{\dom{univ}}{\cod{univ}}$ such that
    \inlineass[name=homomorphism condition]{\conclusion{\foral{
      $\arg[2]{\eq{
        \f{\dom{op}{\va,\vb}}, \cod{op}{\f{\va},\f{\vb}}
      }}$ \comp{for all} $\arg[1]{\inset{\va,\vb}{\dom{univ}}}$.
    }}}
  \end{sdefinition}
\end{mathstructure}
```

Output:

Definition 10.3.2. Let $M_1 = \langle U_1, \circ_1 \rangle$ and $M_2 = \langle U_2, \circ_2 \rangle$ magmas. A **magma homomorphism** is a function $F : U_1 \to U_2$ such that $F(a \circ_1 b) = F(a) \circ_2 F(b)$ for all $a, b \in U_1$.

Now if we instantiate our magma homomorphism:

```
\vardef{vh}[name=H,return={\magmahom[this=H]}]{H}
```

Here is a list of what we can do now:

Syntax	Result
`$\vh!$`	H
`$\vh{}$`	$\langle M_1, M_2, H \rangle$
`$\vh{f}!$`	H
`$\vh{f}{a}$`	$H(a)$
`$\vh{dom}!$`	M_1
`$\vh{cod}{}$`	$\langle U_2, \circ_2 \rangle$
`$\vh{cod}{univ}$`	U_2
`$\vh{dom}{op}!$`	$\circ_1$
`$\vh{cod}{op}{a,b,c}$`	$a \circ_2 b \circ_2 c$

Note how – as one would expect – we can treat `\vh{dom}` and `\vh{cod}` like any other instance of magma.

Note that some of the outputs in the above table are probably not quite what we want. Determining the precise typesetting of an expression involving *nested paths* of fields is difficult, to say the least (e.g., what exactly should `\this` refer to in a deeply nested sequence of fields?).

Using instances within structures is still very useful; at the very least when defining structures. When subsequently *using* structures, however, accessing fields of fields (of fields (of ...)) of an instance should be avoided.

Luckily, there is rarely a need for doing so – in practice, those fields we might want to access in such a way, we usually also want to provide specific notations and talk about independently of the "containing" instance, such that introducing a new variable (or symbol), and assigning the corresponding field to that variable, makes considerably more sense. And subsequently using the variable is easier than concatenating `{...}`, too.

Chapter 11

Complex Inheritance and Theory Morphisms

> We are starting to approach seriously experimental territory.
> While the theory behind all the following is relatively well understood, and their implementation in MMT is mature, the same can not be said out the implementation in sTeX.
> There are still kinks to be ironed out, but feel free to experiment.

We now have all the tools available to progress towards something more interesting. Here is a list of documents with respective modules and symbols we will build on in the following:

`[sTeX/MathTutorial]props/Idempotent.en.tex`

Definition 11.0.1. Let $e \in A$ and $\circ : A \times A \to A$. e is called **idempotent** with respect to $\circ$, if $e \circ e = e$.

Definition 11.0.2. The operation $\circ : A \times A \to A$ is called **idempotent**, if every element $a \in A$ is idempotent with respect to $\circ$.

`[sTeX/MathTutorial]props/Distributive.en.tex`

Definition 11.0.3. Let $\odot : B \times A \to A$ and $\oplus : A \times A \to A$. We say $\odot$ **distributes over** $\oplus$, if $b \odot (a_1 \oplus a_2) = (b \odot a_1) \oplus (b \odot a_2)$ for all $a_1, a_2 \in A$ and $b \in B$.

`[sTeX/MathTutorial]props/Absorption.en.tex`

Definition 11.0.4. Let $\odot : A \times B \to A$ and $\oplus : A \times B \to B$. We say $\odot$ **absorbs**

$\oplus$, if $a_1 \odot (a_1 \oplus b) = a_1$ for all $a_1 \in A$ and $b \in B$.

[sTeX/MathTutorial]algebra/Band.en.tex

Definition 11.0.5. A **band** is an idempotent semigroup.

[sTeX/MathTutorial]algebra/Semilattice.en.tex

Definition 11.0.6. A **semilattice** is a commutative band.

[sTeX/MathTutorial]props/Reflexive.en.tex

Definition 11.0.7. A binary relation R on A is called **reflexive**, if $R(a, a)$ for all $a \in A$.

[sTeX/MathTutorial]props/Symmetric.en.tex

Definition 11.0.8. A binary relation R on A is called **symmetric**, if $R(a, b)$ implies $R(b, a)$ for all $a, b \in A$.

[sTeX/MathTutorial]props/Transitive.en.tex

Definition 11.0.9. A binary relation R on A is called **transitive**, if $R(a, b)$ and $R(b, c)$ implies $R(a, c)$ for all $a, b, c \in A$.

[sTeX/MathTutorial]props/Antisymmetric.en.tex

Definition 11.0.10. A binary relation R on A is called **antisymmetric**, if $R(a, b)$ and $R(b, a)$ implies $a = b$ for all $a, b \in A$.

[sTeX/MathTutorial]orders/Graph.en.tex

Definition 11.0.11. A **directed graph** is a structure $\langle U, R \rangle$, where U is a collection and R a binary relation on U.

Definition 11.0.12. An **(undirected) graph** is a directed graph $\langle U, R \rangle$, where R is symmetric.

[sTeX/MathTutorial]orders/Preorder.en.tex

Definition 11.0.13. A structure $\langle U, \leq \rangle$ is called a **preorder** (or **quasiorder**, or **preordered set**; in short **proset**), if $\leq$ is reflexive and transitive.

[sTeX/MathTutorial]orders/Poset.en.tex

Definition 11.0.14. A preorder $\langle U, \leq \rangle$ is called a **partial order** (or **poset**), if $\leq$ is antisymmetric.

[sTeX/MathTutorial]orders/InfSup.en.tex

Definition 11.0.15. Let $\langle U, \leq \rangle$ a poset. An element $a \in U$ is called an **infimum** or **greatest lower bound** of x_1 and x_2, if $a \leq x_1$, $a \leq x_2$, and for any x with $x \leq x_1$ and $x \leq x_2$, we have $x \leq a$.

Definition 11.0.16. Let $\langle U, \leq \rangle$ a poset. An element $a \in U$ is called a **supremum** or **least upper bound** of x_1 and x_2, if $x_1 \leq a$, $x_2 \leq a$, and for any x with $x_1 \leq x$ and $x_2 \leq x$, we have $a \leq x$.

Note that infima and suprema are more generally defined on *sets* of elements. Doing so in sTeX is significantly more complicated *for now*, and will require some amount of research to make convenient – especially if we want to subsequently define *operators* on pairs of elements, as below. We therefore opt for the simpler version where it is defined as binary from the get go.

[sTeX/MathTutorial]orders/MeetJoinSemilattice.en.tex

Definition 11.0.17. A poset $\langle U, \leq \rangle$ is called a **meet semilattice** if for every two elements a, b the infimum $a \wedge b$ exists.

Definition 11.0.18. A poset $\langle U, \leq \rangle$ is called a **join semilattice** if for every two elements a, b the supremum $a \vee b$ exists.

Definition 11.0.19. An **(order) semilattice** is a meet and join semilattice.

Exercise
Try to implement all of the above yourself!

11.1 Glueing Structures Together

We now want to progress towards lattices, i.e. the following:

Definition 11.1.1. A lattice is a structure $\langle U, \wedge, \vee\rangle$ such that $\langle U, \wedge\rangle$ and $\langle U, \vee\rangle$ are semilattices, and $\vee$ absorbs $\wedge$ and vice versa; i.e. $a\vee(a\wedge b) = a$ and $a\wedge(a\vee b) = a$.

The operations $\wedge$ and $\vee$ are called **meet** and **join**, respectively.

So we make a new module, open an `extstructure` environment and... realize two problems:

1. We can't just extend semilattice: We need *two* copies of semilattice that share a universe, and importing semilattice twice is of course redundant.

2. We also want to *rename* the operations of the two semilattices to be subsequently called `join` and `meet`.

What we need is a way to *inherit* from semilattice while a) *modifying* the symbols therein, and b) not be idempotent – i.e. two imports from the same structure or module should not be identified. We can do that with the `\copymod` macro, which takes three arguments:

1. A *name* for the copy,

2. the structure or module to copy, and

3. a comma-separated list of renamings and redefinitions of the symbol. ⟨*symbol*⟩`=`⟨*def*⟩ redefines ⟨*symbol*⟩, ⟨*symbol*⟩`@`⟨*newname*⟩ renames it, ⟨*symbol*⟩`=`⟨*def*⟩`@`⟨*newname*⟩ (or ⟨*symbol*⟩`@`⟨*newname*⟩`=`⟨*def*⟩) does both.

In our case, we want two copies of semilattice, which we will call `meetsl` and `joinsl`. In the first copy, we only want to rename `op` to `meet`. In the second, we want to rename `op` to `join`, and *also* redefine the universe to be the one from `meetsl`:

```
\copymod{meetsl}{semilattice}{
  op @ meet
}
\copymod{joinsl}{semilattice}{
  univ = \univ,
  op @ join
}
```

You might have already noticed some problem with that – which of the two universes does `\univ` refer to now? (They are *defined* as equal, but LaTeX does not know that!) Or which of the two `commutative axioms` does "`commutative axiom`" refer to now? Everything is ambiguous now!

Not really - if you have wondered why the `\copymod` takes a *name* as argument: The name is prefixed to every symbol name. Hence, the `universe` in `joinsl` is now called `joinsl/universe`, and the one in `meetsl` is called `meetsl/universe`. Furthermore, `\copymod` by default generates no semantic macros for any of the imported symbols – except for those renamed with `@`. In fact, what the `@` syntax actually does, is to generated a semantic macro by that name. If we want to change the *name* (that is shown when using `\symname` et al), we add that new name in square brackets. Hence, what we really want to do is:

```
\copymod{meetsl}{semilattice}{
  univ @ univ,
  op @ [meet]meet
}
\copymod{joinsl}{semilattice}{
  univ = \univ,
  op @ [join]join
}
```

This now gives us two copies of semilattice, generates semantic macros `\univ` for `meetsl/universe`, `\meet` for `meetsl/op` and `\join` for `joinsl/op`, and renames `meetsl/op` to `meet` and `joinsl/op` to `join`.

That allows us to then add the absorption axioms, an `sdefinition` for lattice and subsequently `$\lattice!$` produces $\langle U, \wedge, \vee\rangle$, with all axioms inherited (see `[sTeX/MathTutorial]algebra/Lattice.en.tex`).

11.2 Realizations

A very common situation we find in connection with mathematical structures is that “every *this* is a *that*” (or the conrete case “*this* is a *that*”).

With what we did so far, we are in this situation regarding the algebraic definition of semilattices and the order-theoretic one (exemplary meet semilattice).

In MMT parlance, this corresponds to a total (implicit) theory morphism from “that” to “this”.

In sTeX words, we want to inherit from “that” by assigning all the symbols in “that” to concrete terms. In our case:

`[sTeX/MathTutorial]algebra/SemiLatticeOrder.en.tex`

Definition 11.2.1. Let $\langle U, \circ\rangle$ a semilattice. We let $a \leq b$ iff $a \circ b = a$.

Theorem 11.2.2. *$\langle U, \leq\rangle$ is a meet semilattice.*

Proof: We need to prove the following

reflexivity $a \leq a$: We need to show $a \circ a = a$. Follows from the idempotent axiom.

antisymmetry $a \leq b$ and $b \leq a$ implies $a = b$: Assume $a \circ b = a$ and $b \circ a = b = a \circ b$ (by the commutative axiom). Hence, $a = b$

transitivity If $a \leq b$ and $b \leq c$, then $a \leq c$. : Assume $a \circ b = a$ and $b \circ c = b$ Then $a \circ c = (a \circ b) \circ c = a \circ (b \circ c) = a \circ b = a$. Hence, $a \leq c$.

$a \circ b$ is the infimum of $\{a, b\}$: By definition (and the commutative axiom), $a \circ b \leq a$ and $a \circ b \leq b$. We need to show, that if $x \leq a$ and $x \leq b$, then $x \leq a \circ b$. Assume $x \circ a = x$ and $x \circ b = x$. Then $x \circ (a \circ b) = (x \circ a) \circ b = x \circ b = x$. Hence $x \leq a \circ b$

So to be precise, we want to provide *definientia* for all undefined symbols in meet semilattice (i.e. the relation and meet) and *proofs* for all *axioms* (reflexive axiom, antisymmetric axiom, transitive axiom, and infimum axiom), and by so obtain the fact that every semilattice is a meet semilattice.

For that purpose, we have the `\realize` macro. It behaves like `\copymod`, but does not take a name, and additionally requires that all undefined fields get assigned. So we could do the following:

Example 15

Input:

File [sTeX/MathTutorial]`algebra/SemiLatticeOrder1.en.tex`

```
\begin{extstructure*}{semilattice}
  \realize{meetsl}{
    univ = \univ,
    meet = \op!,
    rel @ [order]order = \map{a,b}{\eq{\op{a,b},a}},
    reflexive axiom = trivial,
    transitive axiom = trivial,
    antisymmetric axiom = trivial,
    infimum axiom = trivial
  }
\end{extstructure*}

\vardef{mysl}[return=\semilattice]{S}
$\mysl{order}{a,b} \qquad \mysl{}[univ,op,order]$
```

Output:

$a \leq_S b \qquad \langle U_S, \circ_S, \leq_S \rangle$

As we can see, we can now access the field `order`, which is renamed from `relation` in meet semilattice and also has the desired definiens in MMT. But of course this approach is very "declarative": We do all the assigning in one macro, rather than narratively as what they *should* be: definitions and proofs.

If we want to achieve the more narrative version at the beginning of the section, we can use the `realization` environment instead. It behaves like the `\realize` macro, but allows us to do the assignments and renamings individuall somewhere in the body of the environment, interleaved with arbitrary text. Additionally, within the environment, all sTeX features that introduce *definientia* (like the `\definiens` macro) induce assignments instead.

To declaratively rename or assign fields, we can then use the `\assign` and `\renamedecl` macros instead. That allows us to do the following instead:

```
\begin{realization}{meetsl}
  \assign{univ}{\univ}
  \assign{meet}{\op!}
  \renamedecl{rel}[order]{order}
  ...
```

...and then use text to do the remaining assignments. For example, we can use the `sdefinition` environment to assign `rel` to the desired definiens:

```
\usestructure{meetsl}
\begin{sdefinition}[for=order]
```

```
    \varbind{va,vb}
    Let $\semilattice![univ,op]$ a \sn{semilattice}.
    We let $\rel{\va,\vb}$
    iff $\definiens{\eq{\op{\va,\vb},\va}}$.
  \end{sdefinition}
```

And now sTeX will use the `\definiens` to assign $a, b \mapsto a \circ b = a$ to the relation of meet semilattice.

Analogously, we can use the `sproof` and `subproof` environments to produce "definientia" (i.e. proofs) for the axioms (see `[sTeX/MathTutorial]algebra/SemiLatticeOrder.en.tex`)

Part III

Extensions for Education

The last two parts have shown generic markup and semantization facilities in sTeX. As said before, investments in semantic markup pay off, iff the impact of a document is high, e.g. if there are many more readers than authors or if the semantic services afforded by the semantic markup can help reduce the help readers need to understand the material.

Educational documents constitute one category of high-impact documents which are supported by the sTeX ecosystem, we will cover them here.

Chapter 12

Slides and Course Notes

TODO[1]

[1] TODO: notesslides.sty

Chapter 13

Problems and Exercises

TODO[1]

[1] TODO: problem.sty

Chapter 14

Exams

TODO[1]

[1] TODO: hwexam.sty